INSTITUTUL
DE
MATEMATICA

INSTITUTUL NATIONAL
PENTRU CREATIE
STIINTIFICA SI TEHNICA

ISSN 0250 3638

QUADRATIC INTERSECTION FORM AND 2-VECTOR BUNDLES ON NONALGEBRAIC SURFACES

by

Vasile BRINZANESCU and Paul FLONDOR

PREPRINT SERIES IN MATHEMATICS
No. 16/1986

BUCURESTI

QUADRATIC INTERSECTION FORM AND 2-VECTOR BUNDLES ON NONALGEBRAIC SURFACES

by

Vasile BRINZANESCU*) and Paul FLONDOR*)

March 1986

*) *Central Institute of Mathematics, str. Academiei 14, 70109 Bucharest, Romania.*

QUADRATIC INTERSECTION FORM AND 2-VECTOR BUNDLES ON NONALGEBRAIC SURFACES

Vasile BRINZANESCU and Paul FLONDOR

Introduction

In the first section of this paper the quadratic intersection form on nonalgebraic surfaces is considered. We show that the quadratic intersection form on the Neron-Severi group of a (smooth, compact, connected) complex surface with algebraic dimension zero is negative definite, modulo torsion (Theorem 1). Then we give a description of this form in the case of algebraic dimension one (Theorem 2).

In the second section some applications to the holomorphic 2-vector bundles on nonalgebraic surfaces are given.

1. The quadratic intersection form on the Neron-Severi group of nonalgebraic surfaces

Let X be a smooth, compact, connected, complex surface and let

$$0 \to \mathbb{Z} \to \mathcal{O}_X \to \mathcal{O}_X^* \to 0$$

be the exact exponential sequence. This sequence gives rise to the exponential cohomology sequence

$$\longrightarrow H^1(X,\mathbb{Z}) \longrightarrow H^1(X,\mathcal{O}_X) \longrightarrow H^1(X,\mathcal{O}_X^*) \longrightarrow H^2(X,\mathbb{Z}) \longrightarrow ,$$

where $H^1(X,\mathcal{O}_X^*) \cong \mathrm{Pic}\, X$ is the Picard group of the surface X. We denote

$$\mathrm{Pic}_o X := \mathrm{Ker}(H^1(X,\mathcal{O}_X^*) \longrightarrow H^2(X,\mathbb{Z}))$$

and we define the Neron-Severi group of the surface X by

$$NS(X) := \mathrm{Im}(H^1(X,\mathcal{O}_X^*) \longrightarrow H^2(X,\mathbb{Z})).$$

We have $NS(X) \cong \mathrm{Pic}\, X/\mathrm{Pic}_o X$.

For nonalgebraic surfaces ($a(X)=0$ or 1) one knows that the quadratic intersection form on the Neron-Severi group is negative semi-definite ([9]). In the case of algebraic dimension zero we have the following result:

<u>Theorem 1. Let X be a complex surface with $a(X)=0$. Then the quadratic intersection form on the Neron-Severi group NS(X) is negative definite, modulo torsion.</u>

<u>Proof</u>. Let $X' \to X$ be the blowing-up of X at a point. Then we have that

$$\mathrm{Pic}\, X' \cong \mathrm{Pic}\, X \oplus \mathbb{Z}\{e\},$$

where $e^2=-1$ and $e.x=0$ for all $x \in \mathrm{Pic}\, X$. It follows that

$$NS(X') \cong NS(X) \oplus \mathbb{Z}\{e\},$$

and the sum is ortogonal. Therefore it is sufficient to prove the statement in the case of a minimal model.

Let X be a minimal model. If the Kodaira dimension Kod(X)= $=-\infty$ we have $b_1(X)=1$ and then, by the Signature Theorem ([2] IV, Theorem 2.13), it follows that the intersection form on $H^{1,1}_{\mathbb{R}}(X)$ is negative definite. Then, its restriction to the subgroup $NS(X) \subset H^2(X,\mathbb{Z})$ is negative definite modulo torsion.

If the Kodaira dimension Kod(X)=0 then X is a K3-surface or a 2-torus. If X is a K3-surface with a(X)=0, then it is well known that the intersection form is negative definite; see for example [5]. The following short argument is from [3]. Let $L \in \mathrm{Pic}\, X$ with $c_1(L)^2=0$. By Riemann-Roch formula we get

$$h^0(L) + h^1(L^*) \geq 2.$$

Since $h^0(M) \leq 1$ for all $M \in \mathrm{Pic}\, X$ as a(X)=0, it follows $h^0(L) \neq 0$ and $h^0(L^*) \neq 0$, hence $L \cong \mathcal{O}_X$ and $c_1(L)=0$.

For the case of a 2-torus with a(X)=0 the detailed proof is given in [6]. We shall present briefly the argument, but first let us remind some general facts about tori.

A complex 2-torus X is isomorphic with $\mathbb{C}^2/\Gamma$, where Γ is a lattice of rank 4 in $\mathbb{C}^2$. One has a natural isomorphism

$$H^2(X,\mathbb{Z}) \cong \mathrm{Alt}^2_{\mathbb{Z}}(\Gamma,\mathbb{Z})$$

of $H^2(X,\mathbb{Z})$ with the space of alternating integer-valued 2-forms on Γ. Let

$$H(\mathbb{C}^2,\Gamma) = \{ H \mid H \text{ hermitian form on } \mathbb{C}^2 \text{ with } \mathrm{Im}\, H(\Gamma \times \Gamma) \subset \mathbb{Z} \} .$$

Since the imaginary part Im H of a hermitian form H is an alternating 2-form which determines completely H, we may consider

$H(\mathbb{C}^2,\Gamma)$ as a subgroup of $\mathrm{Alt}^2_{\mathbb{Z}}(\Gamma,\mathbb{Z}) \cong H^2(X,\mathbb{Z})$. With this identification one has by the theorem of Appell-Humbert (cf. Mumford [11])

$$NS(X) = H(\mathbb{C}^2,\Gamma).$$

Modulo an analytic isomorphism of the 2-torus X, we can take Γ be the lattice generated by the column vectors of the matrix

$$P = \begin{pmatrix} 1 & 0 & p_1 + ip_2 & r_1 + ir_2 \\ 0 & 1 & q_1 + iq_2 & s_1 + is_2 \end{pmatrix} = (I_2,\ B).$$

The matrix P is called the period matrix. We have

$$B_1 = \mathrm{Re}\ B = \begin{pmatrix} p_1 & r_1 \\ q_1 & s_1 \end{pmatrix},\ B_2 = \mathrm{Im}\ B = \begin{pmatrix} p_2 & r_2 \\ q_2 & s_2 \end{pmatrix}$$

and we can choose B such that $B = \det B_2 > 0$.

Consider the complex vector space $\mathbb{C}^2$ as the real vector space $\mathbb{R}^4$ with complex structure given by the matrix

$$J = \begin{pmatrix} 0 & -I_2 \\ I_2 & 0 \end{pmatrix}$$

and take on $\mathbb{R}^4$ also the complex structure given by the matrix

$$J_B = \begin{pmatrix} -B_1B_2^{-1} & -B_2-B_1B_2^{-1}B_1 \\ B_2^{-1} & B_2^{-1}B_1 \end{pmatrix}.$$

Let $f:\mathbb{R}^4 \longrightarrow \mathbb{C}^2$ be the map given by the matrix

$$F = \begin{pmatrix} I_2 & B_1 \\ 0 & B_2 \end{pmatrix}$$

Then $FJ_B = JF$ and since $f(\mathbb{Z}^4) = \Gamma$ the map f extends to an analytic isomorphism between the topological standard torus $\mathbb{R}^4/\mathbb{Z}^4$, with the complex structure given by the matrix J_B, and the complex torus $X = \mathbb{C}^2/\Gamma$.

Now, the Appell-Humbert theorem can be reformulated and we have

$$NS(X) \cong \left\{ A = \begin{pmatrix} A_1 & A_2 \\ -A_2^t & A_3 \end{pmatrix} \in M_4(\mathbb{Z}) \;\middle|\; \begin{array}{l} A \text{ skew-symmetric and} \\ B^tA_1B + A_2^tB - B^tA_2 + A_3 = 0 \end{array} \right\}$$

(see Selder [14]). The condition

$$B^tA_1B + A_2^tB - B^tA_2 + A_3 = 0$$

express the fact that A is the imaginary part Im H of a hermitian form H on $\mathbb{C}^2$. The matrix of the hermitian form in the canonical basis of $\mathbb{C}^2$ is the hermitian matrix

$$H_A = (A_1B_1 - A_2)B_2^{-1} + iA_1.$$

The algebraic dimension of the torus X is given by

$$a(X) = \max\left\{ \text{rank } H_A \;\middle|\; H_A \text{ positive semi-definite} \right\}.$$

Every $A \in NS(X)$ is the first Chern class of a line bundle

$L \in \mathrm{Pic}\, X$ $(A=c_1(L))$. If we identify the group $H^2(X,\mathbb{Z})$ with $\mathrm{Alt}^2_{\mathbb{Z}}(\Gamma,\mathbb{Z})$ then the cup-product on $H^2(X,\mathbb{Z})$ becomes the exterior product of 2-forms (see Mumford [11]). The intersection form on the Neron-Severi group is given by the formula

$$c_1(L).c_1(L')=\alpha\delta'+\alpha'\delta-\beta\gamma'-\beta'\gamma-\theta\tau'-\theta'\tau,$$

where $c_1(L)=A$, $c_1(L')=A'$,

$$A=\begin{pmatrix} A_1 & A_2 \\ -A_2^t & A_3 \end{pmatrix}; A_1=\begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix}, A_2=\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, A_3=\begin{pmatrix} 0 & \tau \\ -\tau & 0 \end{pmatrix} \in M_2(\mathbb{Z})$$

and similarly for A'. For the quadratic intersection form we get:

$$c_1(L)^2 = 2(\alpha\delta-\beta\gamma-\theta\tau).$$

We have the following basic result

$$c_1(L)^2 = 2D.\det H_A.$$

By this formula we get that the quadratic intersection form on the Neron-Severi group $NS(X)$ is negative definite (for details see [6]). As $\mathrm{Kod}(X)\geq 1$ implies $a(X)\geq 1$ the proof is over.

Let now X be a complex surface with $a(X)=1$. One knows that every surface of algebraic dimension 1 is an elliptic fibration. By an elliptic fibration of a surface X one means a proper, connected holomorphic map $f : X \longrightarrow S$, such that the general fibre is a non-singular elliptic curve. Let C be a general fibre of f.

One has $c_1(\mathcal{O}_X(C))^2=C^2=0$. For any $M\in\mathrm{Pic}\,X$ with $c_1(M)^2=0$, the Chern class $c_1(M)$ is ortogonal on NS(X): for if $c_1(L).c_1(M)\neq 0$ for some $L\in\mathrm{Pic}\,X$, then

$$c_1(L\otimes M^{\otimes n})^2 = 2nc_1(L).c_1(M) + c_1(L)^2$$

would be positive for a suitable integer n, contradicting the fact that X is nonalgebraic.

Theorem 2. Let X be a complex surface with $a(X)=1$. Then we have an ortogonal sum $NS(X)/\mathrm{Tors}\,NS(X)=I\oplus N$ such that I is an isotropic subgroup of rank ≤ 1 and the quadratic intersection form is negative definite on the second factor N. Moreover,

(i) if $b_1(X)$ is odd then $I = 0$;

(ii) if X is Kähler then I is generated by a rational multiple of $c = c_1(\mathcal{O}_X(C))$;

(iii) if X has minimal model a K3-surface or a 2-torus then I is generated by $c=c_1(\mathcal{O}_X(C))$.

Proof. Denote by K the lattice $NS(X)/\mathrm{Tors}\,NS(X)$. Let $I=\mathrm{Rad}\,K$ be the radical of K and let $K=I\oplus N$ be a radical splitting (ortogonal sum). If $b_1(X)$ is even it follows, by the Signature Theorem, that the intersection form on $H^{1,1}_{\mathbb{R}}(X)$ is non-degenerate of type $(1,h^{1,1}-1)$. Clearly, $H^{1,1}_{\mathbb{R}}(X)$ has the isotropic index 1, i.e. the maximal isotropic subspace is one dimensional. By the above disscution it follows that the isotropic subgroup I has the rank ≤ 1 and that the intersection form is negative definite on the second factor N. If $b_1(X)$ is odd it follows, by the Signature Theorem, that the intersection form on $H^{1,1}_{\mathbb{R}}(X)$ is non-degenerate of type $(0,h^{1,1})$. Then, obviously, $I=0$ and the intersection form on NS(X) is negative definite,modulo torsion ($c=c_1(\mathcal{O}_X(C))$ is a torsion element). These prove the first statement and also (i).

If X is Kähler ($b_1(X)$ even) it follows that $c=c_1(\mathcal{O}_X(C))\neq 0$ for C a general fibre of the elliptic fibration $f : X \rightarrow S$. Since c is not a torsion element of NS(X), we have (ii).

As for (iii) we can assume f relatively minimal (the fibres free of (-1)-curves). Since NS(X) has no torsion for K3-surfaces and tori we have an ortogonal sum $NS(X)=I \oplus N$; we have to prove that I is generated by c itself. Let $d\neq 0$ be an element of $I(d^2=0)$. Assume that X is K3-surface. Let $L \in$ Pic X such that $d=c_1(L)$. By Riemann-Roch formula we get

$$h^o(L) + h^o(L^*) \geqslant 2;$$

it follows that d (or -d) is effective. Assume that $d=c_1(\mathcal{O}_X(D))$, where D is an effective divisor. Since all curves on X are contained in the fibres of f we have $D=D_1+\ldots+D_n$ with each D_i an effective divisor in a different fibre $(i=1,\ldots,n)$. Obviously $D_i.D_j=0$ for $i\neq j$, hence $D^2=D_1^2+\ldots+D_n^2=0$. From $D_i^2\leqslant 0$ (all i) it follows that $D_i^2=0$ (all i), hence we can suppose that D is contained in a fibre. By Zariski's Lemma it follows that $pD=qX_s$, with $p,q\in\mathbb{Z}$, $p\neq 0$ and X_s a fibre of f $(s\in S)$. One knows that f has no multiple fibres (see [2], p.195), hence p=1 and $D=qX_s$. Since $c_1(\mathcal{O}_X(X_s))=c_1(\mathcal{O}_X(C))=c$ we get that c generates the isotropic subgroup I.

Let now X be a 2-torus. One knows that f has no singular fibres and that, in fact, topologically X is the product $C \times S$. By Künneth formula we have

$$H^2(X,\mathbb{Z})=H^2(\mathbb{C},\mathbb{Z})\oplus(H^1(C,Z)\otimes H^1(S,Z))\oplus H^2(S,\mathbb{Z}),$$

where the subgroup $H^2(C,\mathbb{Z})$ is generated by the Chern class $c=c_1(\mathcal{O}_X(C))$. It follows that the group $H^2(X,\mathbb{Z})/c\mathbb{Z}$ has no torsion, hence the subgroup $I/c\mathbb{Z}$ has no torsion. Then I is generated by c, hence we have (iii).

Remark. In the single remaining case, X properly elliptic with $b_1(X)$ even and non-kählerian (see the Enriques-Kodaira classification [2], VI), we do not know a precise description of the isotropic subgroup I.

2. Filtrable 2-vector bundles on nonalgebraic surfaces

Let E be a holomorphic vector bundle of rank r on a complex surface. The bundle E is called filtrable (cf. Elencwajg-Forster [8]) if there exists a filtration

$$0 = F_o \subset F_1 \subset \ldots \subset F_r = E,$$

with F_i coherent subsheaf of rank i, i=0,1,...,r. While on algebraic surface every holomorphic bundle is filtrable, on nonalgebraic surfaces nonfiltrable bundles exist (see [3], [8], [14], [16]).

As a first application of the previous results we shall prove the following fact:

Proposition 3. Let X be a 2-torus with algebraic dimension zero. A 2-vector bundle E on X is induced by a representation

$$\sigma : \pi_1(X) \longrightarrow GL(2,\mathbb{C})$$

if and only if $c_1(E)=0$ _and_ $c_2(E)=0$.

Proof. We follow [8] Proposition 4.7, where the case $NS(X)=0$ is considered. A bundle induced by a representation of $\pi_1(X)$ possesses an integrable connection, hence all its Chern classes are zero (cf., Atiyah [1]).

Conversely suppose $c_1(E)=0$ and $c_2(E)=0$. Then by [8] Corollary 4.6, E is filtrable. We have two cases:

i) If E is decomposable, it is a sum of two line bundles, $E=L \oplus M$. But $c_1(L) + c_1(M) = c_1(E) = 0$ and $c_1(L).c_1(M) = c_2(E)=0$. It follows that $c_1(L)^2=0$ hence, by Theorem 1, $c_1(L)=0$ and $c_1(M)=0$. Then E is a sum of two topologically trivial line bundles, hence induced by a representation (Appell-Humbert).

ii) If E is indecomposable, we have an exact sequence

$$0 \longrightarrow L \longrightarrow E \longrightarrow M \otimes I_Y \longrightarrow 0,$$

where L, M are holomorphic line bundles and Y is a locally complete intersection of codimension 2 in X or empty. We get $c_1(L) + c_1(M) = c_1(E) = 0$ and $c_1(L) \cdot c_1(M) + \deg Y = c_2(E) = 0$. It follows that $-c_1(L)^2 + \deg Y = 0$, hence $\deg Y = 0$ $(Y = \emptyset)$ and $c_1(L)^2=0$; again by Theorem 1 we have $c_1(L)=0$ and $c_1(M)=0$. If $L \not\cong M$, then $H^0(X,L \otimes M^*)=H^0(X,L^* \otimes M)=0$ because X has no divisors. By Riemann-Roch we get $h^1(X,L \otimes M^*)=0$, contradiction (E is indecomposable). It follows that $L \cong M$ and we have an exact sequence

$$0 \longrightarrow L \longrightarrow E \longrightarrow L \longrightarrow 0.$$

The extensions of L by L are classified by

$$H^1(X, \underline{Hom}(L,L)) \cong H^1(X, \mathcal{O}_X),$$

which has dimension two. The translations operate trivially on $H^1(X, \mathcal{O}_X)$, which shows that E is homogeneous, hence induced by a representation of π_1, [10].

Remark. On an algebraic 2-torus Oda [13] has constructed an indecomposable 2-vector bundle E with $c_1(E)=0$ and $c_2(E)=0$, which is not induced by a representation of π_1.

As for the case of a(X)=1 one also obtaines 2-vector bundles with trivial Chern classes not induced by a representation of π_1.

For the case of decomposable 2-vector bundles take the bundle $E=\mathcal{O}_X(C) \oplus \mathcal{O}_X(-C)$. Then $c_1(E)=0$ and $c_2(E)=-c_1(\mathcal{O}_X(C))^2=0$. Since $c_1(\mathcal{O}_X(C))\neq 0$, $\mathcal{O}_X(C)$ and $\mathcal{O}_X(-C)$ are not induced by a representation of $\pi_1(X)$, hence E is not induced by a representation of $\pi_1(X)$ (C denotes a general fibre of the elliptic fibration $f : X \rightarrow S$).

For the case of indecomposable 2-vector bundles we follow the idea of Oda [13]. Let E′ be an indecomposable 2-vector bundle which is induced by a representation of $\pi_1(X)$. Then by Matsushima [10] and Morimoto [11] it follows that $E'=E_o \otimes L$, where L is a line bundle from $Pic_o(X)$ and E_o is a vector bundle obtained from a unipotent indecomposable representation of $\pi_1(X)$. By Morimoto [11], the bundle $E_o=E' \otimes L^{-1}$ is an extension of the following form:

$$0 \rightarrow \mathcal{O}_X \rightarrow E' \otimes L^{-1} \rightarrow \mathcal{O}_X \rightarrow 0,$$

hence the bundle E′ is an extension of the form

$$(*) \qquad 0 \longrightarrow L \longrightarrow E' \longrightarrow L \longrightarrow 0,$$

with $L \in Pic_o X$ (E' is not simple because there are no simple vector bundles with $c_1=0$, $c_2=0$ on 2-tori; see [13]).

Take now the extensions

$$(**) \qquad 0 \longrightarrow \mathcal{O}_X(C) \longrightarrow E \longrightarrow \mathcal{O}_X(-C) \longrightarrow 0.$$

By Riemann-Roch Theorem we have $h^1(X,\mathcal{O}_X(2C))=h^o(X,\mathcal{O}_X(2C))\neq 0$ and we choose a bundle E which corresponds to a non-zero element of the group $H^1(X,\mathcal{O}_X(2C))$. Remark that $Hom(\mathcal{O}_X(-C),\mathcal{O}_X(C))\neq 0$ and $Hom(\mathcal{O}_X(C),\mathcal{O}_X(-C))=0$. Then, by the Lemma of section 2 in [13], it follows that $End\ E = \mathbb{C} \oplus Hom(\mathcal{O}_X(-C),\mathcal{O}_X(C))$, and in particular E is indecomposable (the first factor consists of the scalar multiplications, while the second factor consists of endomorphisms whose square is zero). We have $c_1(E)=0$, $c_2(E)=0$. Since $Hom(\mathcal{O}_X(-C)$ $\mathcal{O}_X(C))\neq 0$ and $Hom(L,L)\neq 0$ it follows that the extensions (**), (*) are maximal "devissages" of the non-simple, indecomposable 2-bundles E, respectively E'. By Elencwajg and Forster [8], Proposition 1.11, these maximal devissages are uniquely determined, hence the bundle E can not appear in an extension of the type (*). It follows that E is not induced by a representation of $\pi_1(X)$.

Let X be a nonalgebraic 2-torus and let G=NS(X) be the Neron-Severi group of X. If $a \in G$ then we denote by $G_a=a+2G$, the class of a module the subgroup 2G. Let Φ denotes the quadratic intersection form on G and let m_a be the integer

$$m_a := \max_{x \in G_a} \Phi(x).$$

We have the following result: an integer Δ is the discriminant of a filtrable 2-vector bundle E with $c_1(E)=a$ iff it satisfies the conditions $\Delta \leq m_a$, $\Delta \equiv m_a$ (mod 4); see [6]. By using the Theorem 1 (the case of 2-tori) one can compute the bound m_a explicitely (see [6]).

As another application of the Theorem 1 one can obtain the range of Chern classes (c_1, c_2) for simple filtrable 2-vector bundles on nonalgebraic surfaces without divisors (after [3], one knows the range of Chern classes of holomorphic filtrable vector bundles on any nonalgebraic surface). One can show that we obtain the same range as for general filtrable bundles, with some precise exceptions; see [7].

References

[1] M.Atiyah, Complex analytic connections in fibre bundles, Trans. Amer. Math.Soc., 85(1957), 181-207.

[2] W.Barth, C. Peters and A.Van de Ven, Compact complex surfaces, Springer-Verlag, 1984.

[3] C.Bănică et J. Le Potier, Sur l'existence des fibrés vectoriels holomorphes sur les surfaces non-algébriques.

[4] E.Bombieri and D.Husemoller, Classification and embeddings of surfaces, AMS Proc.Symp.Pure Math., 29(1975).

[5] A.Beauville, Surfaces algébriques complexes, Astérisque 54, 1978.

[6] V.Brînzănescu and P.Flondor, Holomorphic 2-vector bundles on nonalgebraic 2-tori, J.reine angew.Math.363(1985),47-58.

[7] V.Brînzănescu and P.Flondor, Simple filtrable 2-vector bundles on complex surfaces without divisors (to appear in Rev.Roumaine Math.Pures Appl.).

[8] G.Elencwajg and O.Forster, Vector bundles on manifolds without divisors and a theorem of deformation, Ann.Inst. Fourier, 32, 4(1982), 25-51.

[9] K.Kodaira, On the structure of compact complex analytic surfaces I, Amer.J.Math.86(1964), 751-798.

[10] Y.Matsushima, Fibrés holomorphes sur un tore complexe, Nagoya Math.J., 14(1959), 1-24.

[11] A.Morimoto, Sur la classification des espaces fibrés vectoriels holomorphes admetant des connexions holomorphes sur un tore complexe, Nagoya Math.J., 15(1959), 83-154.

[12] D.Mumford, Abelian varieties, Oxford Univ.Press, 1970.

[13] T.Oda, Vector bundles on abelian surfaces, Invent.Math., 13(1971), 247-260.

[14] J. Le Potier, Fibrés vectoriels sur les surfaces K3, Séminaire P.Lelong, H.Skoda(Analyse)20e et 21e année 1982/1983.

[15] E.Selder, Picard-Zahlen komplexer Tori, Dissertation Osnabrück, 1981.

[16] H.W.Schuster, Locally free resolutions of coherent sheaves on surfaces, J.reine angew.Math.337(1982), 159-165.