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In [5] E.G.Effros posed the problem of studying inductive limits of C*-algebras of the form $C(X) \otimes M_n$. Because of the complexity of the possible *-homomorphisms $\Phi: C(X) \otimes M_n \longrightarrow C(X) \otimes M_m$ (cf. [3] and [6]) it is reasonable to restrict our attention to specific classes of homomorphisms. In this paper we prove a unicity result concerning inductive limits associated with a sequence of coverings.

A unital homomorphism Φ is called homogeneous if for every $y \in Y$ the subalgebra $\Phi(C(X) \otimes M_n)(y) \subset M_m$ has dimension m/n. (Note that n must divide m since Φ is unital). Suppose that Y has the homotopy type of a finite CW-complex of dimension $\leq 2m/n$ and that $K^\circ(Y)$ has no n-torsion. Then if follows from [3] that there is a (m/n)-fold covering $\psi: Z \rightarrow Y$, a monomorphism $\Phi_1 : C(Z) \otimes M_n \rightarrow C(Y) \otimes M_m$ which satisfies

(0) $$\Phi_1(g \circ \psi \otimes 1_n) = g \otimes 1_m, \qquad g \in C(Y),$$

and a continuous map $\varphi: Z \rightarrow Y$ such that we have the factorization $\Phi = \Phi_1 \circ \varphi^*$.

The homomorphisms Φ_1 satisying equation (0) are called compatible with the covering ψ or ψ-compatible, and they were introduced in [6] for other reasons. The previous decomposition confirms once more their importance, since they are now identified as the nontrivial part of the homogeneous homomorphisms.

We shall consider inductive limits with homomorphisms compatible with some appropriate coverings. Our result is based on a detailed description of such homomorphisms.

An interesting example is supplied by Bunce-Deddens algebras ([2]) which can be described as inductive limits of the form:

$$(1) \qquad \cdots \longrightarrow C(\mathbb{T}^m) \otimes M_{n_i} \longrightarrow C(\mathbb{T}^m) \otimes M_{n_{i+1}} \longrightarrow \cdots$$

where m=1, $\mathbb{T}$ is the unit circle, and the homomorphisms Φ_i are compatible with the coverings $\mathbb{T} \ni z \mapsto z^{n_{i+1}/n_i} \in \mathbb{T}$.

In [6] C.Pasnicu has studied inductive limits of the form (1) with m=2 and he proved that these limits do not depend on the particular choices of the homomorphisms Φ_i compatible with some product coverings $\mathbb{T}^2 \to \mathbb{T}^2$. Moreover, these limits were seen to be isomorphic to tensor products of two Bunce-Deddens algebras.

The aim of this paper is to consider the same problem in an abstract setting.

Given a free action of $\mathbb{T}^m$ on a compact connected manifold X and a strictly increasing sequence of finite subgroups of $\mathbb{T}^m$:

$$G_1 \subset G_2 \subset \ldots G_i \subset G_{i+1} \subset \ldots \subset \mathbb{T}^m$$

we consider inductive limits of the form

$$L = \varinjlim \left(\to C(X_i) \otimes M_{n_i} \xrightarrow{\Phi_i} C(X_{i+1}) \otimes M_{n_{i+1}} \longrightarrow \right)$$

where $X_i = X/G_i$, $n_i = |G_i|$, and the homomorphisms Φ_i are compatible with the coverings $X_i \to X_{i+1}$.

Under some topological restrictions involving the absence of torsion in the cohomology $H^*(X_i, \mathbb{Z})$, we prove that the inductive limit L does not depend on Φ_i and it is isomorphic to the C*-algebra transformation group $C(X) \rtimes G$, where $G = \bigcup_{i=1}^{\infty} G_i$. For the case of Bunce-Deddens algebras this isomorphism was noticed by P. Green.

As a corollary, we extend the result from [6] to the m-dimensional torus.

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1. PRELIMINARIES

We shall denote by M_n the C*-algebra of $n \times n$ complex matrices and by 1_n its unit.

Suppose that X is a compact, connected, real manifold and let S be a finite group acting freely on X.

If $k = |S|$ (the order of S), then the quotient map onto the orbit space $\psi : X \to X/S$ is a regular k-fold covering. Let n be a positive integer.

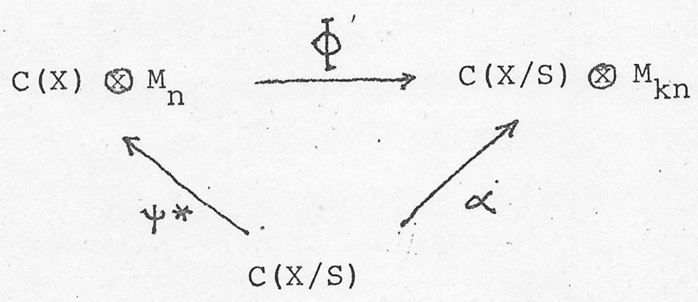
We recall from [6] that a unital homomorphism

$$\phi : C(X) \otimes M_n \to C(X/S) \otimes M_{kn}$$

is called compatible with the covering ψ or ψ-compatible if

$$(2) \qquad \phi\,(g \circ \psi \otimes 1_n) = g \otimes 1_{kn}, \qquad g \in C(X/S).$$

Looking at the following diagram



where $\psi^*(g) = (g \circ \psi) \otimes 1_n$ and $\alpha(g) = g \otimes 1_{kn}$, it is clear that ψ-compatible homomorphisms may be viewed as a kind of sections for the fibering $X \to X/S$.

Assume that the K-theory group $K^\circ(X)$ is torsion-free and that $\dim(X) \leq 2k$. Then it follows from [3, thm 1.3] that $\phi = v(\phi' \otimes id_{M_n})v^*$, for some unitary $v \in C(X/S) \otimes M_{kn}$ and some

ψ-compatible homomorphism $\Phi' : C(X) \to C(X/S) \otimes M_k$. Moreover, it is proved in [3] that there is a continuous map $p: X \to P(C^k) =$ = the space of all one dimensional self-adjoint projections acting on $\mathbb{C}^k$, such that Φ' is given by the following formula

(3) $$\Phi'(f)(\psi(x)) = \sum_{s \in S} f(s.x)\, p(s.x), \quad f \in C(X),\ x \in X.$$

Of course since Φ' is unital we must have

(4) $$\sum_{s \in S} p(s.x) = 1_n, \qquad x \in X.$$

Also, it is clear that both Φ and Φ' are monomorphisms. Despite the previous description, we don't know a priori if ψ-compatible homomorphisms (or equivalently maps $p: X \to P(\mathbb{C}^k)$ satisfying (4)) do exist.

However, if we assume that S is an abelian group and that the second cohomology group $H^2(X/S, \mathbb{Z})$ is torsion-free, then such homomorphisms can be constructed as follows. By [3] there is a continuous map $u: X \to U(k) \simeq U(l^2(S))$ such that $u(s.x) =$ $= \rho(s)\, u(x)$, $x \in X$, $s \in S$, where $\rho: S \to U(k)$ is the right regular representation of S. Now if $\{e_s^\circ\}_{s \in S}$ are the orthogonal projections onto the subspaces $[\delta_s]$ spanned by the vectors in the canonical basis $\{\delta_s\}_{s \in S}$ of $l^2(S)$, then the homomorphism $\Phi' : C(X) \to C(X/S) \otimes M_k$ given by

(5) $$\Phi'(f)(\psi(x)) = u(x)^* \Big(\sum_{s \in S} f(s.x)\, e_s^\circ\Big) u(x), \quad f \in C(X),\ x \in X$$

is compatible with the covering $X \to X/S$. Note that Φ' is well defined since $\rho(s)^* e_t^\circ \rho(s) = e_{ts}^\circ$, $t, s \in S$. Consider now the crossed-product C*-algebra

$$C(X) \rtimes S = \{ F \in C(X) \otimes M_k : F(s.x) = \rho(s)F(x)\rho(s)^*, \ x \in X, s \in S \}$$

Then the unitary u can be used to give an isomorphism $H: C(X) \rtimes S \rightarrow C(X/S) \otimes M_k$. To see this, we identify $C(X/S) \otimes M_k$ with

$$\{ F \in C(X) \otimes M_k : F(s.x) = F(x), \text{ for all } x \in X, s \in S \},$$

and we take $H(F) = u^*Fu$. Note that if

$$j: C(X) \rightarrow C(X) \rtimes S, \quad j(f)(x) = \sum_{s \in S} f(s.x) e_s^{\circ},$$

is the canonical embedding, then the isomorphism H is such that $H \circ j = \phi'$.

2. INNER EQUIVALENCE

Assume $S \subset T^m$ and also that the action of S on X is induce by a continuous free action of T^m on X.

Then we are able to give a more complete description of homomorphisms Φ which are compatible with the covering $X \to X/S$. Our description will imply that any two such homomorphisms are inner equivalent.

Lemma 1.1. Let $x_0 \in X$, let $p: X \to P(\mathbb{C}^k)$ be a continuous map which satisfies equation (4) and assume that $H^2(X, Z)$ is torsion-free. Then there is a continuous map $u: X \to U(k)$ such that

(6) $$p(s.x) = u(x)^* p\ (s.x_0) u(x) \qquad x \in X,\ s \in S\ .$$

Proof. Set $e_s(x) = p(s.x)$. Then $\{e_s\}_{s \in S}$ is a partition of the unity in the C*-algebra $C(X) \otimes M_k$. Since the action of S on X is induced by a continuous action of T^m which is a pathwise connected space, it follows that the projections $\{e_s\}_{s \in S}$ are mutually equivalent in $C(X) \otimes M_k$. To see this let $a: [0,1] \to T^m$ be a continuous path from 1 to s. Then $e_{a(t)}(x) = p(a(t).x)$, $t \in [0,1]$ is a continuous path of projections from e_1 to e_s. By a standard argument we find now a partial isometry $e_{s,1} \in C(X) \otimes M_k$ such that $e^*_{s,1} e_{s,1} = e_1$ and $e_{s,1} e^*_{s,1} = e_s$. Define $e_{s,s} = e_s$ and $e_{s,t} = e_{s,1} e^*_{t,1}$ to obtain a system of matrix units in $C(X) \otimes M_k$. Now consider the C*-homomorphisms $\Phi, \Phi_0: M_k \to C(X) \otimes M_k$ given by

$$\Phi_0\ (e_{s,t}(x_0)) = 1_{C(X)} \otimes e_{s,t}(x_0)$$

$$\Phi\ (e_{s,t}(x_0)) = e_{s,t}$$

(Note that in the above definition we identified M_k with the C^*-algebra generated by $\{e_{s,t}(x_0)\}_{s,t\in S}$) Since the complex line bundles on X are classified by $H^2(X, Z)$ which we suppose to be torsion-free, it follows form [3, prop.1.1] that there is some unitary $u \in C(X) \otimes M_k$ such that $\phi_0 = u\phi u^*$. This implies that $e_s(x_0) = u(x)\ e_s(x)\ u(x)^*$. Hence $p(s.x)=u(x)^*p(s.x_0)u(x)$ for all x in X and $s \in S$.

□

Let $\rho : S \to B(l^2(S))$ be the right regular representation of S. We identify $B(l^2(S))$ with $C^*\langle e_{s,t}(x_0):s,t \in S\rangle \simeq M_k$, so that $\rho(r)^*\ e^0_{s,t}\ \rho(r) = e^0_{sr,tr}$. As an easy consequence of the equation (6) we obtain that every $u(s.x)u(x)^*\rho(s)^*$ commutes with all $e_{t,t}(x_0)$, $t \in S$. Setting $w_s(x)=u(s.x)u(x)^*\ \rho(s)^*$ it follows that w_s is diagonal with respect to the projections $e_{t,t}(x_0)$. More precisely there are continuous functions $w(t,s):X \to \mathbb{T}$, $s,t \in S$, such that

$$w_s(x) = \sum_{t\in S} w_{t,s}(x)\ e_{t,t}(x_0)\quad , \qquad x \in X. \tag{7}$$

Moreover, it follows from the definition of $\{w_s\}_{s\in S}$ that $w_s(t.x) = w_{st}(x)\ \rho(s)w_s(x)^*\ \rho(s)^*$.

Then we have corresponding relations for $w_{t,s}$:

$$w_{t,sr}(x) = w_{ts,r}(x)\ w_{t,s}(r.x) \qquad r,s,t \in S, x \in X. \tag{8}$$

Equations (8) look like some "cocycle relations". Our next task is to resolve the "cocycle $(w_{s,t})$" i.e. to find continuous maps $d_s:X \to T$, $s \in S$ such that

$$w_{s,t}(x)=d_{ts}(x)d_t(s.x)^{-1} \qquad x \in X,\ s,t \in S. \tag{9}$$

Suppose now that the maps $\langle d_s \rangle_{s \in S}$ have been found and set

$$v(x) = \Big(\sum_{s \in S} d_s(x) e_{s,s}(x_0) \Big) \; u(x).$$

Then an easy computation shows us that

$$(10) \qquad v(s.x) = \rho(s) v(x) \qquad\qquad x \in X, \; s \in S \text{ and}$$

$$(11) \qquad p(x) = v(x)^* \, p(x_0) \, v(x), \qquad\qquad x \in X$$

To make clear the proof we choose to resolve the cocycle $(w_{s,t})$ in an abstract setting. For technical reasons we make the following :

Definition 2.2. A finite abelian group S is said to have the property (H) if given any six-tuple $E=(A, \sigma, D, \alpha, w, (b, d_b))$ consisting of:

1) a free transitive action σ of S on a set A

$A \times S \ni (a,s) \longmapsto a.s \in A$,

2) an action α of S by automorphisms, on an abelian group D,

3) a cocycle $w : A \times S \to D$ satisfying,

(12) $w(a,st) = w(a.s,t) \, \alpha_t(w(a,s))$, $\qquad a \in A, \; s,t \in S.$

4) a couple $(b, d_b) \in A \times D$,

there is a map $d : A \to D$ such that

(13) $d(b) = d_b$ and $w(a,s) = d(a.s) \, \alpha_s(d(a))^{-1}$, $a \in A, \; s \in S$

Lemma 2.3. The cyclic group Z_n has the property (H).

Proof. Let $Z_n = \langle 1, s, \ldots, s^{n-1} \rangle$. From (12) we get

$$\alpha_{s^{-k}}(w(a.s^k, s)) = w(a.s^k, s^{-k+1}) \, w(a.s^{k+1}, s^{-k})^{-1}$$

and consequently

$$(14)\quad \prod_{k=0}^{n-1} \alpha_{s^{-k}} (w(a.s^k,s)) = 1 \qquad a \in A.$$

Since S acts transitively on A it follows that A=b.S. To define the map $d:A \to D$ we put $d(b) = d_b$ and then we find recursively $d(b.s^k) \in D$, $1 \leq k \leq n-1$ such that

$$(15)\quad d(b.s^{k+1}) = w(b.s^k,s)\ \alpha_s(d(b.s^k)),\ 0 \leq k \leq n-1.$$

Combining (14) and (15) we get that the formula

(15) holds even if k=n.

Therefore we have now proved that

$$(16)\quad w(a,s) = d(a.s)\ \alpha_s(d(a))^{-1} \qquad \text{for all } a \in A.$$

Let $t=s^k$ and assume that

$$(17)\quad w(a,t)=d(a,t)\ \alpha_t(d(a))^{-1} \qquad \text{for all } a \in A.$$

Since by (12) we have

$$w(a,st)=w(a.s,t)\ \alpha_t(w(a,S))$$

we infer from (16) and (17) that

$$w(a,st)=d(a,st)\ \alpha_t(d(a.s))^{-1}\ \alpha_t(d(a.s))\ \alpha_{ts}(d(a))-1=$$
$$=d(a.st)\ \alpha_{st}(d(a))^{-1}$$

The assertion follows now by induction.

<u>Lemma 2.4.</u> If both the groups G and S have the

property (H) then the direct sum $R=G \oplus S$ has the property (H).

Proof. Think G and S as subgroups of R. Given a R-six-tuple $E=(A, \sigma, D, \alpha, w, (b, d_b))$ as in Definition 2.2, we apply the property (H) of S with respect the S-six-tuple:

$$E|_S = (b.S, \sigma|_S, D, \alpha|_S, w|_{b.S\times S}, (b, d_b)) .$$

Therefore we obtain a map $d': b.S \to D$ that satisfies

(18) $\qquad d'(b)=d_b$ and $w(b.s,t)=d'(b.st)\,\alpha_t(d'(b.s))^{-1}, s,t \in S.$

To extend d′ to an appropiate map on A, note first that $A = \bigcup_{s \in S} b.sG$ and then apply the property (H) of G, for each $s \in S$, relative to the G-six-tuple

$$E_S=(b.sG, \sigma|_G, D, \alpha|_G, w|_{b.sG\times G'} \quad (b.s, d'(b.s))).$$

In this way we obtain a map $d: A \to D$ which extends d′ and such that

(19) $\qquad w(a,g)=d(a.g)\,\alpha_g(d(a))^{-1}$ $\qquad$ for a in A and $g \in G$.

Since R=S.G, to complete the proof, it remains to show that the map d satisfies all needed relations:

(20) $\qquad w(a,sg)=d(a.sg)\,\alpha_{sg}(d(a))^{-1}$ $\qquad a \in A,\ s \in S,\ g \in G.$

It is convenient to prove first (20) with g=1. If $a \in A$ then we may write a in a unique way as a=b.th with $t \in S$ and $h \in G$. Since we can derive from (12):

$$w(b.th,s)\ \alpha_s(w(b.t,h)=w(b.ts,h)\ \alpha_h(w(b.t,s))$$

(both terms beeing equal to w(b.t,sh)) it follows that

$$w(a,s)=w(b.ts,h)\ \alpha_h(w(b.t,s))\ \alpha_s(w(b.t,h)^{-1}),$$

so that using (18) and (19) we get

$$w(a,s)=d(b.tsh)\ \alpha_h(d(b.ts))^{-1}\ \alpha_h(d(b.ts))\ \alpha_{hs}(d(b.t))^{-} \tag{21}$$

$$.\ \alpha_s(d(b.th))^{-1}\ \alpha_{sh}(d(b.t))=d(a.s)\ \alpha_s(d(a))^{-1}$$

Since $w(a,sg)=w(a.s,g)\ \alpha_g(w(a,s))$

(20) can be derived from (19) and (21) as in the proof of Lemma 2.3.

<u>Corollary 2.5</u>. The finite abelian groups have the property (H).

<u>Proof</u>. Since the finite abelian groups are direct sums of cyclic groups our Corollary is a straightforward consequence of the previous lemmas.

Assume that $H^2(X/S,\mathbb{Z})$ and $H^2(X,\mathbb{Z})$ are torsion free.

<u>Theorem 2.6.</u> If $\Phi : C(X) \to C(X/S) \otimes M_k \simeq C(X/S) \otimes B(l^2(S)$ is compatible with the covering $X \to X/S$, then there is some continuous unitary valued map $u:X \to U(k)$ such that

$$u(s.x)=\rho(s)u(x) \qquad x \in X,\ s \in S \tag{22}$$

$$\Phi(f)(\psi(x))=u(x)^*(\sum_{s\in S} f(s.x)e_s^\circ)u(x) \tag{23}$$

Proof. Let $w=(w_{s,t})$ be the "cocycle" that appeared in the discussion before Definition 2.2. Let S acts on S by translations $\tau_s(t)=ts$, let $D=C(X,T)$ and let define an action of S on D by setting $\alpha_s(f)(x)=f(s.x)$, $s \in S$, $f \in D$. Applying the property (H) of S relative to the six-tuple $E=(S,\tau,D,\alpha,w,(1,1))$ if follows that the cocycle $(w_{s,t})$ can be resolved, so that the description of Φ is given by (3), (10) and (11).

Corollary 2.7. Any two C*-homomorphisms $\Phi,\Psi : C(X) \to C(X/S) \otimes M_k$ compatible with the covering $X \to X/S$ are inner equivalent i.e. there is some unitary $v \in C(X/S) \otimes M_k$ such that $\Phi = v\Psi v^*$.

Proof. Theorem 2.6. provides us descriptions of Φ and Ψ with appropriate unitaries u and u_1. After conjugating with an unitary in $C(X/S) \otimes M_k$, we may suppose that these descriptions are given relative to the same projections $\{e_s^\circ\}$ $s \in S$.

Consequently we may choose $v=u_1^*u$ since

$$v(s.x)=u_1(s.x)^*u(s.x)=u_1(x)^*\rho(s)^*\rho(s)u(x)=v(x)$$

and it is clear from (23) that $\Psi = v\Phi v^*$. ☒

A negative Example. Let S^2 be the two-sphere and $S^2/\mathbb{Z}_2 \simeq \mathbb{P}^2$ be the two-dimensional real projective space. Since $H^2(\mathbb{P}^2,\mathbb{Z}) = \mathbb{Z}_2$ Corollary 2.7 doesn't apply. In fact it can be proved that there are infinitely many homomorphisms $C(S^2) \longrightarrow C(\mathbb{P}^2) \otimes M_2$ compatible with the canonical covering $S^2 \longrightarrow \mathbb{P}^2$ which are not inner equivalent.

3. SOME INDUCTIVE LIMITS

As in the previous section we start with a continuous free action of T^m on a compact, connected, real manifold X. Let

$$G_1 \subset G_2 \subset \cdots \subset G_i \subset G_{i+1} \quad \cdots$$

be an infinite tower of finite subgroups of T^m. Let $n_i = | G_i |$, $k_i = | G_{i+1}/G_i |$ and note that $n_{i+1} = n_i k_i$. If X_i denotes the quotient space X/G_i, we have a natural k_i-fold covering $X_i \to X_{i+1}$ whose deck-group S_i is isomorphic to G_{i+1}/G_i.

In this section we deal with inductive limits of the form:

$$(24) \qquad \cdots \to C(X_i) \otimes M_{n_i} \xrightarrow{\Phi_i} C(X_{i+1}) \otimes M_{n_{i+1}} \to \quad \cdots$$

where each homomorphism Φ_i is compatible with the covering $X_i \to X_{i+1}$.

The main result is the following:

<u>Theorem 3.1.</u> Assume that the manifolds X_i have no torsion in cohomology i.e. $H^*(X_i, Z)$ is torsion-free for any $i \geq 1$.

Then the inductive limit $\lim (C(X_i) \otimes M_{n_i}, \Phi_i)$ does not depend on the particular choice of the homomorphisms Φ_i. In fact it depends only on the group $G = \bigcup_{i=1}^{\infty} G_i$ since it is isomorphic to the crossed product C^*-algebra $C(X) \rtimes G$.

<u>Proof.</u> As a first step we prove that any two homomorphisms

By considering a refinement of the sequence in (24) we may assume that $\dim X_i \leq n_{i+1}/n_i$

ϕ_i, $\Psi_i : C(X_i) \otimes M_{n_i} \longrightarrow C(X_{i+1}) \otimes M_{n_{i+1}}$ are inner equivalent. Recall that $n_{i+1} = n_i k_i$. Since $H^*(X_{i+1}, \mathbb{Z})$ is torsion free, it follows from [1] that $K^\circ(X_{i+1})$ is torsion free. Hence, by the results quoted in section 1, we may assume that $n_i = 1$. At this point the assertion follows from Corollary 2.7. To conclude the first part of the theorem we recall Lemma 2.1 of [5] which asserts that the inductive limits $\lim (A_i, \phi_i)$ and $\lim (A_i, \Psi_i)$ are isomorphic if the homomorphisms Φ_i and Ψ_i are inner equivalent.

To proceed further, let us consider the diagram

$$(25) \qquad \begin{array}{ccc} \cdots \to C(X) \rtimes G_i & \xrightarrow{\quad J_i \quad} & C(X) \rtimes G_{i+1} \to \cdots \\ \Big\downarrow H_i & & \Big\downarrow H_{i+1} \\ \cdots \to C(X_i) \otimes M_{n_i} & \xrightarrow{\quad \Phi_i \quad} & C(X_i) \otimes M_{n_{i+1}} \to \cdots \end{array}$$

where (J_i) are the canonical embeddings, (H_i) are the isomorphisms described in section 1, and (Φ_i) are chosen such that $\Phi_i = H_{i+1} J_i H_i^{-1}$. With this definition it is straighforward to check that the homomorphisms Φ_i are compatible with the coverings $X_i \to X_{i+1}$. Since the inductive limit of the upper row in the diagram (25) is equal to

$$C(X) \rtimes G = \Big(\bigcup_{i=1}^{\infty} C(X) \rtimes G_i \Big)^- ,$$

it turns out that the unique limit that arise from the diagram (24) is isomorphic to $C(X) \rtimes G$.

Let $\mathbb{T}^m$ act on $X = \mathbb{T}^m$ by translations. Given a finite subgroup S of $\mathbb{T}^m$ it is well known that $\mathbb{T}^m / S \simeq \mathbb{T}^m$. Further, since

$H^*(T, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}$, it follows by Künneth formula that $H^*(T^m, \mathbb{Z})$ is torsion free. Therefore we may apply Theorem 3.1 to obtain a unicity result concerning the inductive limits of the form

$$(26) \qquad \dots \to C(T^m) \times M_{n_i} \xrightarrow{\Phi_i} C(T^m) \otimes M_{n_{i+1}} \to \dots$$

where the homomorphisms Φ_i are compatible with (n_{i+1}/n_i)-fold coverings $T^m \to T^m$. Moreover if these coverings correspond to the tower of subgroups

$$G_1 \subset G_2 \subset \dots \subset T^m \quad ,$$

and we assume that $G = \bigcup_{i=1}^{\infty} G_i$ is dense in T^m, then it can be proved that the C*-algebra $C(T^m) \rtimes G$ is simple and it has a unique faithful trace state.

Suppose now that the homomorphisms Φ_i are compatible with the coverings

$$(27) \qquad (z_1, \dots z_m) \longmapsto (z_1^{P_1(i)}, \dots, z_m^{P_m(i)})$$

and let $n_k(i) = \prod_{j=1}^{i} P_k(j)$, $1 \leq k \leq m$. Let $A(n_k)$ be the Bunce-Deddens algebra associated with the generalized integer $n_k = (n_k(i))_{i \geq 1}$. Then we have the following Corollary which extends the main result of [6].

Corollary 3.2. The inductive limit (26) does not depend on the choice of the homomorphisms Φ_i compatible with the coverings (27). Moreover it is isomorphic to the C*-tensor product $\bigotimes_{k=1}^{m} A(n_k)$.

Proof. We apply Theorem 3.1 with

$$G_i = G_1(i) \times G_2(i) \times \ldots \times G_m(i);$$

where

$$G_k(i) = \{ z \in \mathbb{T} : z^{n_k(i)} = 1 \}$$

Let $G_k = \bigcup_{i=1}^{\infty} G_k(i)$ and note that $G = G_1 \times G_2 \times \ldots \times G_m$. If we denote by L the unique limit arising from (26) then

$$L = C(\mathbb{T}^m) \rtimes G \simeq \bigotimes_{k=1}^{m} C(\mathbb{T}) \rtimes G_k$$

and

$$C(\mathbb{T}) \rtimes G_k \simeq A(n_k).$$

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