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1. INTRODUCTION

In the present paper we consider acceleration waves (a.w.) which are propagating in an elastoviscoplastic (e.v.p.) body with instantaneous plasticity (i.p.) subjected to a large deformation.

The elastoviscoplastic behaviour of the body in a fixed material point X (in the framework of thermodynamics with internal variables) was described by Teodosiu [1], Mandel [2,3], Halphen [4], Dafalias [5], Loret [6], and it was based on the concept of relaxed (unstressed or natural) configuration which was developed by Lee, Lin [7,3]. The deformation gradient is multiplicatively decomposed into its elastic, F^e, and its plastic part, F^p. Both these tensors are described by some constitutive and evolution equations in terms of the second Piola-Kirchhoff stress tensor, $\mathcal{T}$, with respect to the relaxed configuration. The state of materials depends on a certain set of internal variables, which are introduced by some evolution equations.

When a body $\mathcal{B}$ undergoes an inelastic deformation, it generally has not a global natural configuration and therefore Soos [9] has introduced the concept of current relaxed isoclinic configuration in the description of the e.v.p. behaviour of the body at a fixed material point X. This concept is based on the local configuration of the material point which was elaborated by Noll [10,11].

Briefly in the second section we recall the basical assumptions[12] of the approach to thermoelastoviscoplastic body considered here. The behaviour of the body at $X\in\mathcal{B}$ is described with respcect to a current local relaxed configuration, K_{xt}, which is an equivalence class of all configuration coinciding in a neighborhood of X (see Noll [10,11]).

The propagation conditions for the a.w. in e.v.p. body as well as the properties of the acoustic tensor for an acceleration waves are obtained in third section. If a stress state of the body corresponds to a unloading or to a neutral process, or it is inside of the current yield surface, then the acoustic tensor results symmetric and it is similar to those obtained by Wang and Truesdell [13] but the measures of the deformation is the elastic part of the deformation gradient F.

In the forth section we prove the existence of the acoustic tensor for any e.v.p. state of the body and we give the condition in which the acoustic tensor becomes symmetric. The existence of the plastic potential (with respect to the variable $\Sigma \equiv (F^e)^T F^e \mathcal{T} \equiv C^e \mathcal{T}$ - used by Halphen [4], Teodosiu, Sidoroff [14], Halphen and Nguyen [15]) which will be either made plausible by analysing the microstural rearrangements (see Mandel [2] - in the case of small elastic deformation; Teodosiu, Sidoroff [14]) or simply postulated by Halphen [4], leads to the symmetry of the acoustic tensor.

All consideration in the fifth section refere to the case of e.v.p. body with i.p. when elastic deformations are small. If the elastic constitutive equation is invertible with respect to elastic Cauchy-Green tensor, C^e, then the acoustic tensor is symmetric if and only if the associated plastic flow low is assumed relative to the instantaneous plastic term only. In the simplest case when the elastic constitutive equation is isotropic and linear we analyse the propagation of the longitudinal and transverse a.w.

The a.w. which are propagating in an elastic body subjected to large strain were considered by Wang and Truesdell [13] and similar problem for a viscoplastic body, described by some rate constitutive equation, was considered by Suliciu [16].

The following notation will be used:

$\mathcal{E}$ - a three-dimensional euclidian space with the translation vector space $\mathcal{V}$, Lin = $\{ A : \mathcal{V} \rightarrow \mathcal{V}$ linear$\}$, Sym = $\{ A \in$ Lin, $A = A^T \}$, where A^T is the transpose of A, Invlin $\subset$ Lin - the set of all invertible linear mappings; Psym - the set of all positive symmetric mappings, Orth = $\{ Q \in$ Lin, $QQ^T = I \}$, I - the identity tensor,

$a \otimes b$ and $A \otimes B$, and so on, the tensor product of $a,b \in \mathcal{V}$ and $A,B \in Lin$ respectively; $a \cdot b$, $A \cdot B = tr\, AB^T$ - the scalar product of $a,b \in \mathcal{V}$ and $A,B \in Lin$ respectively; $|A| = \sqrt{trAA^T}$; $K_X = \{ \gamma / \gamma$ configuration of $\mathcal{B}$, $\nabla (\gamma \circ K^{-1})/_{K(X)} = I \}$ - a local configuration, i.e. the equivalence class defined by the configuration K; ∇ -the gradient field; K_{Xt} a local configuration of a fixed X at the moment t, which can be identified (see Noll [10,11]) with the invertible linear transformation of $\mathcal{T}_X$ (the tangent space in X) onto $\mathcal{V}$; $\partial_c h(C,\alpha)$-the partial derivative of h with respect to C; if h is a scalar valued function then $\partial_c h(C,\alpha) \in Lin$, for $C \in Lin$; $\{A\}_s$, $\{A\}_a$- the symmetric and the antisymmetric parts of the tensor A, respectively.

2. ELASTOVISCOPLASTIC CONSTITUTIVE EQUATIONS

Let $\mathcal{B}$ be a thermoelastoviscoplastic body with instantaneous plasticity. The behaviour of the body at X will be described in terms of the following constitutive assumptions [12]:

A.1. For any (χ, θ), where $\chi : \mathcal{B} \times R \rightarrow \mathcal{E}$ is the motion of the body and $\theta : \mathcal{B} \times R \rightarrow (0, +\infty)$ is the temperature field, there exist: $K_{Xt} \in Invlin\, (\mathcal{T}_X, \mathcal{V})$ a current local relaxed configuration and $\alpha_{K_{xt}} \equiv \alpha$ - the set of internal variables, such that:

A.2. The thermoelastic constitutive equation is of the form:

$$\mathcal{T} \equiv \mathcal{T}_{K_{xt}} = 2 \rho_{K_{xt}} \partial_{C^e} \psi_{K_{xt}} (C^e, \theta, \alpha) \equiv h_{K_{xt}} (C^e, \theta, \alpha) \tag{1}$$

where ψ is the specific Helmholtz energy and

$$C^e = (F^e)^T F^e, \quad \text{with } F^e \equiv F^e_{K_{xt}} = \nabla \chi (X,t) K^{-1}_{xt}, \tag{2}$$

is the left Cauchy-Green elastic strain tensor; $\mathcal{T}_{K_{xt}}$ is the Piola-Kirchhoff stress tensor related to T (the Cauchy stress tensor) by the relation

$$\mathcal{T} = (\det F^e)(F^e)^{-1} T (F^e)^{-T} \tag{3}$$

and $\rho_{K_{xt}}$ is the density relativ to K_{xt} defined by:

$$\tilde{\varrho} \equiv \varrho_{K_{xt}} = \varrho \det F^{e} \tag{4}$$

with $\varrho = \varrho(x,t)$, where $x = \chi(X,t)$, the actual mass density.

A.3. K_{xt} and $\alpha_{K_{xt}}$ are given by the following evolution equations:

$$\dot{F}^{p}(F^{p})^{-1} = A_{K_{xt}}(\pi,\theta,\alpha) + \langle \lambda_{K_{xt}} \rangle B_{K_{xt}}(\pi,\theta,\alpha) \tag{5}$$

$$\dot{\alpha} = l_{K_{xt}}(\pi,\theta,\alpha) + \langle \lambda_{K_{xt}} \rangle m_{K_{xt}}(\pi,\theta,\alpha)$$

where F^{p} is the plastic tensor relative to the reference configuration k of the body and is defined by:

$$F^{p} \equiv F^{p}_{K_{xt}} = K_{xt}(\nabla k(X))^{-1}; \tag{6}$$

F^{p} and α satisfy the initial condition $F^{p}(0) = F^{p}_{0}$, $\alpha(0) = \alpha_{o}$.

The instantaneous plastic functions $B_{K_{xt}}$ and $m_{K_{xt}}$ are assumed to be zero iside the current yield surface S(t) which is defined by the plastic function $\mathcal{F}_{K_{xt}}$ as follows:

$$S(t) = \{ (\pi,\theta) \in \mathrm{Sym} \times R / \mathcal{F}_{K_{xt}}(\pi,\theta,\alpha) = 0 \}.$$

We associate, say for instance, the function A_{k}

$$A_{k}(Z, F^{p}) = A_{K_{xt}}(Z) \tag{7}$$

to the function $A_{K_{xt}}$ in order to specify the dependence on K_{xt}. Further the configuration k will be omited. We suppose that all constitutive and evolution functions are continuous on their arguments, but $\mathcal{F}$ and h have all the first order partial derivatives continuous.

On S(t) the following consistency conditions

$$1 + m \cdot \partial_{\alpha}\mathcal{F} + \partial_{F^{p}}\mathcal{F} \cdot B(F^{p})^{T} = 0, \quad \partial_{\alpha}\mathcal{F} \cdot 1 + \partial_{F^{p}}\mathcal{F} \cdot AF^{p} = 0 \tag{8}$$

hold.

Here $\lambda_{K_{xt}}$ -the plastic loading factor is defined on S(t) by the relation

$$\lambda \equiv \lambda_{K_{xt}} = \partial_{\Pi} \mathcal{F} \cdot \dot{\Pi} + \partial_{\theta} \mathcal{F} \dot{\theta} \qquad (9)$$

and $\langle \lambda \rangle = \frac{1}{2}(\lambda + |\lambda|)$.

Let us (for instance) denote by $\tilde{\mathcal{F}}$ the composition of $\mathcal{F}$ with the elastic constitutive function h:

$$\tilde{\mathcal{F}}(C^e, \theta, \alpha, F^p) = \mathcal{F}_k(h_k(C^e, \theta, \alpha, F^p), \theta, \alpha, F^p) \qquad (10)$$

A.4. On S(t) the plastic loading factor λ is defined in a unique way by (χ, θ).

Under the last assumption it was proved the existence of the <u>complementary plastic factor</u>

$$\beta_{K_{xt}} = (F^p)^{-1} \partial_{C^e} \tilde{\mathcal{F}} (F^p)^{-T} \cdot \dot{C} + \partial_\theta \tilde{\mathcal{F}} \dot{\theta} + \partial_\alpha \tilde{\mathcal{F}} \cdot \tilde{\ell} + \partial_{F^p} \tilde{\mathcal{F}} (F^p)^T \cdot \tilde{A} - 2 \partial_{C^e} \tilde{\mathcal{F}} \cdot \{C^e \tilde{A}\}_s \qquad (11)$$

(with $C = F^T F$), which has the same sign as $\lambda_{K_{xt}}$, such that:

$$\langle \lambda_{K_{xt}} \rangle = 1/\gamma \langle \beta_{K_{xt}} \rangle . \qquad (12)$$

Here γ -<u>the hardening parameter</u> is defined on S(t) by

$$\gamma = 2 \partial_{C^e} \tilde{\mathcal{F}} \cdot \{C^e \tilde{B}\}_s - \partial_\alpha \tilde{\mathcal{F}} \cdot \tilde{m} - \partial_{F^p} \tilde{\mathcal{F}} \cdot \tilde{B} F^p > 0 \qquad (13)$$

and γ is positive.

By using the complementary plastic (which has the meaning of the rate of the current yield surface in the deformation space) it follows that (F^p, α) are solutions of some differential equations with initial data for any given (χ, θ).

The relations $(2)_2$ and (6) involve the multiplicative decomposition of the deformation gradient relative to k into its elastic and plastic parts:

$$F \equiv \nabla \chi(X, t)(\nabla k(X))^{-1} = F^e F^p \qquad (14)$$

From (4) and (14) it results

$$\tilde{\rho} = \rho_o / |\det F^p| \tag{15}$$

where ρ_o is the initial mass density.

In what follows we suppose that the functions relative to the reference configuration k, see formula (7), are not dependent on F^p, i.e. they obey the "time invariance condition", and that the specific Helmholtz energy function is expressed [see Teodosiu [1970], Mandel [1971]) by:

$$\psi_{K_{xt}}(C^e, \theta, \alpha) = \psi(C^e, \theta) + \bar{\psi}(\theta, \alpha) \tag{16}$$

From (1) and (16) we get following form of the thermoelastic constitutive equation:

$$\tilde{\pi} = 2\tilde{\rho}\, \partial_{C^e} \psi(C^e, \theta) \equiv h_k(C^e, \theta, \alpha, F^p) \tag{17}$$

Therefore the function $\tilde{\mathcal{F}}$, introduced by (10), depends on F^p only through $\tilde{\rho} = \rho_o/\det\ F^p$, i.e.

$$\tilde{\mathcal{F}}(C^e, \theta, \alpha, F^p) = \tilde{\mathcal{F}}_k(2\tilde{\rho}\, \partial_{C^e} \psi(C^e, \theta), \theta, \alpha). \tag{18}$$

From (13), togather with (18) and (8) we obtain the following expression of the hardening parameter:

$$\gamma = 4\tilde{\rho}\, \partial_{\tilde{\pi}} \mathcal{F} \cdot \partial^2_{C^e} \psi\{C^e B\}_s + (\tilde{\pi} \cdot \partial_{\tilde{\pi}} \mathcal{F})\operatorname{tr} B + 1 > 0, \tag{19}$$

using the formula

$$\partial_{\tilde{\pi}} \mathcal{F} \cdot 2\tilde{\rho}\, \partial^2_{C^e} \psi(M) = \partial_{C^e} \tilde{\mathcal{F}} \cdot M, \quad \text{for all } M \in \mathrm{Sym}, \tag{20}$$

derived from (18).

In that follows the processes (χ, θ) will be supposed to be isotermic, i.e. $\theta = \theta_o$, where θ_o is the initial temperature.

3. THE PROPAGATION CONDITIONS

Most of the notation and definitions used are as in Truesdell, Noll [19] and

Wang, Truesdell [13].

The acceleration wave (a.w.) is a regular surface $\bar{\mathcal{S}}_t$ in $k(\mathcal{B})$: $\bar{\phi}(X,t) = 0$ with the property that

$$\nabla\bar{\phi}(X, t) \neq 0 \text{ on } \bar{\phi}(X,t) = 0 \tag{21}$$

which is singular with respect to the second derivatives of the motion χ. The image of the a.w. in $\chi(\mathcal{B}, t) \equiv \mathcal{B}_t$, denoted by $\mathcal{S}_t$, is characterized by

$$\phi(x,t) = \bar{\phi}(\chi^{-1}(x,t),x) . \tag{22}$$

We denote the unit normal of $\bar{\mathcal{S}}_t$ and $\mathcal{S}_t$ by n and $\bar{n}$ respectively. Then

$$\bar{n} = \frac{\nabla\bar{\phi}}{|\nabla\bar{\phi}|} \quad , n = \frac{\nabla\phi}{|\nabla\phi|} . \tag{23}$$

From (23) and (22) it follows that

$$n = F^{-T}\bar{n} \cdot \frac{|\nabla\bar{\phi}|}{|\nabla\phi|} . \tag{24}$$

The normal speeds $\bar{u}$ and u are introduced by

$$\bar{u} = -\frac{\partial\bar{\phi}}{\partial t}\Big/|\nabla\bar{\phi}| \quad , u = -\frac{\partial\phi}{\partial t}\Big/|\nabla\phi| \tag{25}$$

If we introduce the intrinsec speed of $\mathcal{S}_t$, U:

$$U = u - v \cdot n \tag{26}$$

where v is the velocity of the material point X at time t on $\mathcal{S}_t$ then

$$\bar{u} = U\frac{|\nabla\phi|}{|\nabla\bar{\phi}|} \tag{27}$$

Since the first derivatives of χ are continuous on the surface $\bar{\mathcal{S}}_t$ at all t there exists a vectors field $\bar{s}$, defined on $\bar{\mathcal{S}}_t$ and called the amplitude vector of the a.w. in the configuration $\bar{\mathcal{S}}_t$ such that (see Wang, Truesdell [13], Truesdell, Toupin [20]):

$$[\nabla F(X,t)] = \bar{s} \otimes \bar{n} \otimes \bar{n}, \; [\dot{F}(X,t)] = -\bar{u}\bar{s} \otimes \bar{n}, \; [(\partial^2 \chi / \partial t^2)(X,t)] = \bar{u}^2 \bar{s} \qquad (28)$$

Here [f] denotes the jump of f on the surface $\bar{\mathcal{S}}_t$.

We suppose that F^p and α are continuous on the wave. Then from (14) we see that F^e is also continuous since [F] = 0. From elastic constitutive equation (17) with (2) and (4) we get $[\mathcal{T}] = 0$ and therefore $[\pi_o] = 0$, where π_o is the Piola-Kirchhoff stress tensor relative to k and defined by:

$$\pi_o = \det F^p (F^p)^{-1} \pi (F^p)^{-T} \quad \text{or} \quad \pi_o = (\det F) F^{-1} T F^{-T} \qquad (29)$$

Since $\bar{\mathcal{S}}_t$ is not singular with respect to π_o there exists $\tilde{\underset{\sim}{\pi}} \in$ Sym such that

$$[\operatorname{div} \pi_o] = \tilde{\underset{\sim}{\pi}} \bar{n}, \quad [\partial \pi_o / \partial t] = -\bar{u} \, \tilde{\underset{\sim}{\pi}} \qquad (30)$$

The jump condition for $\dot{F}^p$ and $\dot{\alpha}$ follow from the evolution equations (5):

$$[\dot{F}^p] = h(\lambda)(BF^p \otimes \partial_\pi \mathcal{F})[\dot{\pi}], \; [\dot{\alpha}] = h(\lambda)(m \otimes \partial_\pi \mathcal{F})[\dot{\pi}] \qquad (31)$$

where $h(\lambda)$ is the Heavyside function: $h(\lambda) = 1$ if $\lambda > 0$ and $h(\lambda) = 0$ if $\lambda \leq 0$. We note that we have no jumps for $\dot{F}^p$ and $\dot{\alpha}$ in the case of thermoelastoviscoplastic body, i.e. when B and m are every where zero. We can obtain the jump $[\dot{F}^e]$ replacing $(28)_2$ and $(31)_1$ in (14) derived with respect to t:

$$\dot{F} = \dot{F}^e F^p + F^e \dot{F}^p \qquad (32)$$

Then

$$[\dot{F}^e] = -\bar{u}(\bar{s} \otimes \bar{n})(F^p)^{-1} - h(\lambda)(F^e B \otimes \partial_\pi \mathcal{F})[\dot{\pi}] \qquad (33)$$

Now differentianting the elastic constitutive equation (17) with respect to t we obtain the following relation between the jumps of $\dot{C}^e$ and $\dot{\pi}$:

$$\{I_4 + h(\lambda) \operatorname{tr} B(\pi \otimes \partial_\pi \mathcal{F})\}[\dot{\pi}] = 2 \tilde{\rho} \, \partial^2_{C^e} \psi \, [\dot{C}^e] \qquad (34)$$

Here I_4 is unit forth order tensor. We can employ the jump condition (33) in

$$[\dot{C}^e] = [\dot{F}^e](F^e)^T + (F^e)^T[\dot{F}^e]$$

and

$$[\dot{C}^e] = -2\bar{u}\,\{(F^p)^{-T}(\bar{n}\otimes\bar{s})F^e\}_s - 2h(\lambda)(\{C^eB\}_s\otimes\partial_{\mathcal{T}}\mathcal{F})[\dot{\mathcal{T}}] \tag{35}$$

follows.

If we introduce the forth order tensor $\mathcal{L}_o$ defined by

$$\mathcal{L}_o = I_4 + h(\lambda)\operatorname{tr} B(\mathcal{T}\otimes\partial_{\mathcal{T}}\mathcal{F}) + 4h(\lambda)\tilde{\varrho}(\partial^2_{C^e}\psi\{C^eB\}_s)\otimes\partial_{\mathcal{T}}\mathcal{F} \tag{36}$$

then from (34) and (35) we obtain

$$\mathcal{L}_o[\dot{\mathcal{T}}] = -4\bar{u}\,\tilde{\varrho}\,\partial^2_{C^e}\psi\,\{(F^p)^{-T}(\bar{n}\otimes\bar{s})F^e\}_s \tag{37}$$

If $\mathcal{L}_o$ is an invertible for the order tensor for any elastoplastic state, then

$$[\dot{\mathcal{T}}/\tilde{\varrho}] = -4\bar{u}\,\mathcal{L}_1\partial^2_{C^e}\psi\,\{(F^p)^{-T}(\bar{n}\otimes\bar{s})F^e\}_s, \text{ with } \mathcal{L}_1 = \mathcal{L}_o^{-1}. \tag{38}$$

holds.

From the balance low of impulse (in the reference configuration)

$$\operatorname{div}(F\mathcal{T}_o) + \varrho_o b_o = \varrho_o\ \partial^2\chi/\partial t^2)$$

when the body force b_o is continuous, we obtain <u>the dynamic compatibility condition:</u>

$$\varrho_o\bar{u}^2\bar{s} - \mathcal{T}_o\cdot(\bar{n}\times\bar{n})\bar{s} - F\underset{\sim}{\mathcal{T}}\,\bar{n} = 0 \tag{39}$$

The jump $[\dot{\mathcal{T}}_o] = -\bar{u}\,\underset{\sim}{\mathcal{T}}$ can be expressed in terms of $[\dot{\mathcal{T}}]$ since the tensors $\mathcal{T}_o$ and $\mathcal{T}$ are related by $(29)_1$. Taking the derivative with respect to t in $(29)_1$ and using $(3.1)_1$ we obtain

$$-(\bar{u}/\varrho_o)\underset{\sim}{\mathcal{T}} \equiv [\dot{\mathcal{T}}_o/\varrho_o] \equiv (F^p)^{-1}\mathcal{L}_2[\dot{\mathcal{T}}/\tilde{\varrho}](F^p)^{-T} \tag{40}$$

where

$$\mathscr{L}_2 = I_4 + h(\lambda)\mathrm{tr}\, B(\mathscr{T} \otimes \partial_{\mathscr{T}} \mathscr{F}) - 2h(\lambda)\{B\mathscr{T}\}_s \otimes \partial_{\mathscr{T}} \mathscr{F} \qquad (41)$$

We introduce (38) into (40) and since $\bar{u} \neq 0$ it follows that:

$$F\tilde{\mathscr{L}}\bar{n} = 4\rho_o F^e(\mathscr{L}_2 \mathscr{L}_1 \partial^2_{C^e}\psi)\{(F^p)^{-T}(\bar{n} \otimes \bar{s})F^e\}_s (F^p)^{-T}\bar{n} \qquad (42)$$

We use (42) into (39) and we obtain the propagation condition in the reference configuration $k(\mathscr{B})$:

$$0 = \rho_o \bar{u}^2 \bar{s} - \mathscr{T}_o \cdot (\bar{n} \otimes \bar{n})\bar{s} - 4\rho_o F^e \mathscr{L}_2 \mathscr{L}_1 \partial^2_{C^e}\psi\{(F^p)^{-T}(\bar{n} \otimes \bar{s})F^e\}_s (F^p)^{-T}\bar{n} \qquad (43)$$

We can express the condition (43) in the actual configuration. Using (24) we obtain

$$\mathscr{T}_o \cdot (\bar{n} \otimes \bar{n}) = (\det F)(|\nabla\phi| / |\nabla\bar{\Phi}|)^2 T \cdot (n \otimes n) \qquad (44)$$

with $(29)_2$ and

$$\{(F^p)^{-T}(\bar{n} \otimes \bar{s})F^e\}_s = (|\nabla\phi| / |\nabla\bar{\Phi}|)\{(F^e)^T n \otimes (F^e)^T \bar{s}\}_s \qquad (45)$$

Introducing (44) and (45) into (43) we get the propagation condition in actual configuration:

$$\rho U^2 s - T \cdot (n \otimes n)s - 4\rho F^e(\mathscr{L}_2 \mathscr{L}_1 \partial^2_{C^e}\psi)\{(F^e)^T n \otimes (F^e)^T s\}_s (F^e)^T n = 0 \qquad (46)$$

since U - the intrinsec speed of the a.w. is expressed by $\bar{u}$ with (27) and s-the amplitude vector of the a.w. in the actual configuration is defined by

$$s = (|\nabla\phi| / |\nabla\bar{\Phi}|)^2 \bar{s} \qquad (47)$$

The acoustic tensor (for a similar notion see for instance [13] or [16] is defined for all v B by:

$$Q(n)v = T \cdot (n \otimes n)v + 4\rho F^e(\mathscr{L}_2 \mathscr{L}_1 \partial^2_{C^e}\psi)\{(F^e)^T n \otimes (F^e)^T v\}_s (F^e)^T n \qquad (48)$$

and the propagation condition for the a.w. in e.v.p. body with i.p. becomes

$$\varrho U^2 s - Q(n)s = 0 \qquad (49)$$

In the next section we shall prove the existence of Q(n).

In the case of e.v.p. body we obtain an equivalent Fresnel-Hadamand theorem:

The amplitude s of the a.w. travelling in the direction n must be a proper vector of the acoustic tensor Q(n); the correspondig proper number is ϱU^2; where U is the intrinsec speed of the wave. The acoustic tensor Q(n) given by (48), with $(38)_2$, (41) is determined by the elastic constitutive equation (17), (2)-(4), the elastic part of deformation, F^e, the wave-normal n and also by B-the instantaneous plastic function and $\partial_{\Pi}\mathcal{F}$ -the normal to S(t) when the stress Π lies on the current yield surface, S(t).

If the elastic properties of the material depend on α (i.e. $\psi_{K_{xt}}$ can not be expressed by (16)) then Q(n) will contain also the instantaneous function m from the evolution equation of $\dot{\alpha}$ (see $(5)_2$) and α will be involved not only in B and $\partial_{\Pi}\mathcal{F}$ but also in $\partial^2_{C^e}\psi$.

4. THE ACOUSTIC TENSOR

In this section we shall prove the existence of the acoustic tensor Q(n) and we shall analyse its symmetry.

Let F^e, F^p, α, T characterise the e.v.p. state of the body in X and at time t for a given motion χ and the temperature θ_0.

Two cases must be considered:

a) the process is such that $\mathcal{F}(\Pi, \theta_0, \alpha) < 0$ or $\mathcal{F}(\Pi, \theta_0, \alpha) = 0$ but $\lambda \leq 0$. Then $B(\Pi, \theta_0, \alpha) = 0$ and $h(\lambda) = 0$ respectively.

b) the process corresponds to a loading, i.e. $\mathcal{F}(\Pi, \theta_0, \alpha) = 0$ and $\lambda > 0$.

In the first case, Q(n)-the acoustic tensor for the wave normal n is defined for any $v \in \mathcal{V}$ by:

$$Q(n)v = T\cdot(n \otimes n)v + 4\varrho F^e \partial^2_{C^e}\psi \{(F^e)^T n \otimes (F^e)^T v\}_S (F^e)^T n \equiv Q^e(n)v \qquad (50)$$

since $\mathcal{L}_2$ and $\mathcal{L}_o$ given by (41) and (36) respectively are the identity. It results $\mathcal{L}_1 = \mathcal{L}_o^{-1} = I_4$. The existence of $Q^e(n)$ follows from the linearity in v of the right side of (50). In the case (a) the acoustic tensor Q(n) has been denoted by $Q^e(n)$ since it corresponds to the elastic part of deformation F^e. The results is similar to those presented by Wang and Truesdell [13] but F^e is measured from the plastically deformed configuration K_{xt}.

b) Let $\mathcal{T}$ be on the current yield surface and $\lambda > 0$. We prove that the linear $\mathcal{L}_o$: Sym $\rightarrow$ Sym, defined by

$$\mathcal{L}_o X = X + ((\mathrm{tr}\, B)\mathcal{T} + 4\tilde{\rho}\, \partial^2_{C^e} \psi(\{C^e B\}_s))\, \partial_{\mathcal{T}} \mathcal{F} \cdot X \equiv Y \tag{51}$$

is an invertible forth order tensor. We take the scalar product of (51) with $\partial_{\mathcal{T}} \mathcal{F}$ and we obtain

$$\gamma\, \partial_{\mathcal{T}} \mathcal{F} \cdot X = \partial_{\mathcal{T}} \mathcal{F} \cdot Y \tag{52}$$

with the hardening parameter $\gamma > 0$ given by (19). Then $\partial_{\mathcal{T}} \mathcal{F} \cdot X = 0$ if and only if $\partial_{\mathcal{T}} \mathcal{F} \cdot Y = 0$. Now we replace (52) into (51) and finally we get

$$X = \{I_4 - 1/\gamma\, (\mathrm{tr}\, B)\, \mathcal{T} \otimes \partial_{\mathcal{T}} \mathcal{F} \;\; + 4\tilde{\rho}\, \partial^2_{C^e} \psi(\{C^e B\}_s)\}\, Y \tag{53}$$

Therefore

$$X = \mathcal{L}_1 Y \text{ with } \mathcal{L}_1 = I_4 - (h(\lambda)/\gamma)((\mathrm{tr}\, B)\mathcal{T} + 4\tilde{\rho}\, \partial^2_{C^e} \psi\, \{C^e B\}_s) \otimes \partial_{\mathcal{T}} \mathcal{F} \tag{54}$$

In this way the inversability of $\mathcal{L}_o$ given by (36) has been proved for any elastoplastic state of the body.

By direct calculus we derive the formula

$$\mathcal{L}_2 \mathcal{L}_1 = I_4 - (2h(\lambda)/\gamma)(\{B\mathcal{T}\}_s + 2\tilde{\rho}\, \partial^2_{C^e} \psi\, \{C^e B\}_s) \otimes \partial_{\mathcal{T}} \mathcal{F} \tag{55}$$

From (48) with (55) and (50) we obtain the following formula for Q(n):

$$Q(n)v = Q^e(n)v - (2h(\lambda)/\gamma\, \det F^e)(\partial_{\mathcal{T}} \mathcal{F} \cdot 4\tilde{\rho}\, \partial^2_{C^e} \psi(\{(F^e)^T n \otimes (F^e)^T v\}_s) F^e(\{B\mathcal{T}\}_s + + 2\tilde{\rho}\, \partial^2_{C^e} \psi(\{C^e B\}_s))(F^e)^T n \tag{56}$$

Another expression of $Q(n)$

$$Q(n)v = Q^e(n)v - (4h(\lambda)/\gamma \det F^e)(\partial_{C^e}\tilde{\mathcal{F}} \cdot \{(F^e)^T n \otimes (F^e)^T v\}_s) F^e(\{B\pi\}_s + + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s))(F^e)^T n \tag{57}$$

can be obtained if we use (18) and (20). The existence of the acoustic tensor results at once as the second term in (57) is linear too, with respect to v, for all $v \in \mathcal{V}$.

PROPOSITION 4.1. The acoustic tensor $Q(n)$ is symmetric for all $n \in \mathcal{V}$ if and only if there exists a scalar valued function ν such that

$$\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s) = \nu\,\partial_{C^e}\tilde{\mathcal{F}}. \tag{58}$$

Proof. $Q(n)$, for a certain n, is symmetric if and only if

$$\begin{aligned}(\partial_{C^e}\tilde{\mathcal{F}} \cdot \{(F^e)^T n \otimes (F^e)^T v\}_s)(\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s))(F^e)^T n \cdot (F^e)^T w = \\ (\partial_{C^e}\tilde{\mathcal{F}} \cdot \{(F^e)^T n \otimes (F^e)^T w\}_s)(\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s))(F^e)^T n \cdot (F^e)^T v\end{aligned} \tag{59}$$

holds for all $v, w \in \mathcal{V}$, since $Q^e(n)$ define by (50) is symmetric for a given wave-normal n. Here $\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s) \in \mathrm{Sym}$ and the first factor in (59) can be transformed by using the formula $A \cdot (x \otimes y) = A^T y \cdot x$ written for $A = \partial_{C^e}\tilde{\mathcal{F}} \in \mathrm{Sym}$. If we put

$$a = F^e(\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s))(F^e)^T n;\ b = F^e\,\partial_{C^e}\tilde{\mathcal{F}}\,(F^e)^T n \tag{60}$$

into (59), then $(a \cdot v)(b \cdot w) = (a \cdot w)(b \cdot v)$. The last relation holds for all $v, w, \in \mathcal{V}$ if and only if a is paralel with b. Therefore there exists a scalar valued function depending on C^e, $\theta_0, \alpha, \tilde{\rho}$ and n such that

$$F^e(\{B\pi\}_s + 2\tilde{\rho}\,\partial^2_{C^e}\psi(\{C^e B\}_s))(F^e)^T n = \nu\, F^e\,\partial_{C^e}\tilde{\mathcal{F}}\,(F^e)^T n \tag{61}$$

The last equality takes place for any $n \in \mathcal{V}$ if and only if (58) holds with independent of n, since $F^e \in \mathrm{Invlin}$.

PROPOSITION 4.2. The acoustic tensor $Q(n)$ is symmetric for all wave-normal n if and only if the instantaneous plastic function $B(\pi, \theta_0, \alpha)$ satisfies:

$$(\partial_{C^e}\Sigma)A \cdot B = \nu \partial_{C^e}\tilde{\mathcal{F}} \cdot A$$

where Σ is the non-symmetric tensor defined by:

$$\Sigma = C^e \pi \tag{63}$$

Proof. Let we consider the function

$$C^e \in \mathrm{Sym} \longrightarrow \Sigma = C^e \pi \equiv 2\tilde{\rho}\, C^e \partial_{C^e}\psi \in \mathrm{Lin} \tag{64}$$

The differential with respect to C^e of the above function is given by:

$$(\partial_{C^e}\Sigma)(A) = A\pi + C^e \partial_{C^e}\pi\,(A) \tag{65}$$

for any $A \in \mathrm{Sym}$. By taking the scalar product of (58) with any arbitrar $A \in \mathrm{Sym}$ and by using the symmetry of the forth order tensor $\partial_{C^e}\pi \equiv 2\tilde{\rho}\,\partial^2_{C^e}\psi$ we obtain condition (62).

REMARKS

1. The non-symmetric tensor Σ plays a special role for the inelastic (plastic) deformation being the cofactor of $L^p = F^e(\dot{F}^p(F^p)^{-1})(F^e)^{-1}$ in the plastic power $\Sigma \cdot L^p$ (see Teodosiu, Sidoroff [14], Halphen, Nguyen [15], Halphen [4]).

2. The existence of the plastic potential:

$$B = \partial_{\Sigma}\tilde{\phi}(\Sigma, \theta, \alpha, \tilde{\rho}), \text{ with } \Sigma = C^e \pi \tag{66}$$

postulated by Halphen [4] leads to the symmetry of the acoustic tensor and moreover

$$\partial_{C^e}\tilde{\phi}(\Sigma, \theta, \alpha, \tilde{\rho}) = \nu \partial_{C^e}\tilde{\mathcal{F}}(C^e, \theta, \alpha, \tilde{\rho}). \tag{67}$$

5. ACCELERATION WAVES IN THE CASE OF SMALL ELASTIC DEFORMATION

All considerations in this section refere to elastoviscoplastic body with instantaneous plasticity but:

i) the elastic deformation are small, i.e.

$$C^e = I + 2\varepsilon^e \quad \text{or} \quad U^e = I + \varepsilon^e \tag{68}$$

with

$$|\varepsilon^e| = \sqrt{\mathrm{tr}(\varepsilon^e)^2} << 1$$

and finite rotation $R^e \in$ Orth.

Piola-Kirchhoff stress tensor $\tilde{\pi}$ will be related to Cauchy stress tensor T by

$$\tilde{\pi} = (R^e)^T T R^e \tag{69}$$

ii) the elastic constitutive function is linear in deformations:

$$\tilde{\pi} = \mathcal{E}\varepsilon^e = 2\tilde{\rho}\,\partial_{C^e}\psi \tag{70}$$

with $\mathcal{E}$ a forth order symmetric invertible tensor. From (69), $(68)_2$ and (70) we obtain an equivalent form of the elastic constitutive equation:

$$T = \bar{\mathcal{E}}\bar{\varepsilon}^e, \text{ where } \bar{\varepsilon}^e = R^e\varepsilon^e(R^e)^T, \quad V^e = I + \bar{\varepsilon}^e \tag{71}$$

with the following relation between tensors $\mathcal{E}$ and $\bar{\mathcal{E}}$:

$$\bar{\mathcal{E}}A = R^e\mathcal{E}((R^e)^T A R^e)(R^e)^T \tag{72}$$

written for all $A \in$ Sym.

In the case of isotropic linear elastic constitutive equation we have

$$T = \bar{\lambda}^e(\mathrm{tr}\,\varepsilon^e)I + 2\bar{\mu}^e\varepsilon^e \tag{73}$$

where $\bar{\lambda}^e$ and $\bar{\mu}^e$ are the Lame elastic constants.

P.5.1. Under the conditions i), ii) the acoustic tensor Q(n) is in Sym for all $n \in \mathcal{V}$ if and only if

$$\{B\}_s = \nu\,\partial_{\tilde{\pi}}\mathcal{F} \tag{74}$$

and the symmetric acoustic tensor is expressed by

$$Q(n)v = (\bar{\mathcal{E}}\{n \otimes v\}_s)n - (h(\lambda)/\gamma)\ (\bar{\mathcal{E}}\partial_T\mathcal{F} \otimes \bar{\mathcal{E}}\,\partial_T\mathcal{F})\{n \otimes v\}_s n \tag{75}$$

with

$$\gamma = 1 + \nu\,\partial_T\mathcal{F}\cdot\bar{\mathcal{E}}\partial_T\mathcal{F} + (T\cdot\partial_T\mathcal{F})\,\mathrm{tr}\,\partial_T\mathcal{F} \tag{76}$$

and

$$\bar{\mathcal{F}}(T,\theta,\bar{\alpha},R^e) = \mathcal{F}((R^e)^T T R^e,\theta,\alpha) \equiv \mathcal{F}(\pi,\theta,\alpha) \quad (77)$$

and $\bar{\alpha} = (R^e\alpha_j(R^e)^T, \alpha_k)$-the actual internal variables, when α_j (with $j\in\{\overline{1,n_1}\}$) are tensors and α_k (with $k\in\{\overline{1,n_2}\}$) scalars.

Proof. In the case of small elastic deformations the tensor $\Sigma \simeq \pi \in \mathrm{Sym}$ and from (62) we get

$$(\partial_{C^e}\pi)A\cdot\{B\}_s = \nu\,\partial_{C^e}\tilde{\mathcal{F}}\cdot A \quad (78)$$

for all $A\in\mathrm{Sym}$. By using (20), (70) with $(68)_1$ in (78) we obtain

$$\mathcal{E}A\cdot\{B\}_s = \nu\,\partial_{\pi}\mathcal{F}\cdot\mathcal{E}A \quad (79)$$

with $\mathcal{E}$-the forth order invertible tensor. (74) results at once. Now ν is considered as depending on π,θ,α since $\varepsilon^e = \mathcal{E}^{-1}\pi$.

The corresponding form of $Q(n)$ is calculated from (57) with (50) in the form

$$Q(n)v = ((\bar{\mathcal{E}}\varepsilon^e)\cdot(n\otimes n))v + (\bar{\mathcal{E}}\{n\otimes v\}_s)n -$$

$$- (\nu h(\lambda)/\gamma)(1/\det F^e)\ \mathcal{E}\partial_{\pi}\mathcal{F}\cdot\{(R^e)^T n\otimes(R^e)^T v\}_s R^e(\mathcal{E}\partial_{\pi}\mathcal{F})(R^e)^T n \quad (80)$$

by using (70) - (72) in (50) and (70) with (20) in (58). With (70) for $\bar{\alpha}$ given by $(77)_2$ the plastic function $\mathcal{F}(\pi,\theta,\alpha)$ becomes (77). It follows that

$$\partial_T\bar{\mathcal{F}} = R^e\,\partial_{\pi}\mathcal{F}\,(R^e)^T \quad (81)$$

By using the symmetry of $\mathcal{E}$, and the definite on of $\bar{\mathcal{E}}$ given by (72) in (80) we obtain (75) since the first term in (80) can be neglected in the presence of the second one, when elastic deformations are small. The expression (19) of the hardening parameter becomes

$$\gamma = \nu\{\mathcal{E}\partial_{\pi}\mathcal{F}\cdot\partial_{\pi}\mathcal{F} + (\pi\cdot\partial_{\pi}\mathcal{F})\,\mathrm{tr}\,\partial_{\pi}\mathcal{F}\} + 1 \quad (82)$$

with (70) and (74). Further we use (81) and (72) in (82) and the expression (76) follows at once.

P.5.2. 1) For a given e.v.p. state and for each wave-normal n, $Q(n)\in\mathrm{Sym}$

satisfies the condition $Q(n)n \cdot n > 0$ if and only if the scalar constitutive function given by (75) obeis

$$\inf_{n \in \mathcal{V}, |n|=1} \bar{\mathcal{E}} \cdot (n \otimes n \otimes n \otimes n) \geq$$

$$\geq (h(\lambda)\nu/1 + \nu(\partial_T \bar{\mathcal{F}} \cdot \bar{\mathcal{E}} \partial_T \bar{\mathcal{F}} + T \cdot \partial_T \bar{\mathcal{F}} \operatorname{tr} \partial_T \bar{\mathcal{F}})) \sup_{n \in \mathcal{V}, |n|=1} |(\bar{\mathcal{E}} \partial_T \bar{\mathcal{F}})n \cdot n|^2 \qquad (83)$$

2) If condition (83) holds for a given e.v.p. state then at least one wave-normal n allows a longitudinal amplitude.

Proof

1) The proof follows at once from (75) in which v is replaced by n and with calculated from (76).

2) The assumption leads to $Q(n)n \cdot n > 0$ for all $v \in \mathcal{V}$. Following Wang and Truesdell (13) we observe that $n \to Q(n)n/|Q(n)n|$ maps continuously the unit sphere into itself and it maps no point into its antipode.

Such map has as fixed point. Thus there exists $n_e \in \mathcal{V}$, $|n_e| = 1$, such that $Q(n_e)n_e = |Q(n_e)n_e|n_e$, i.e. n_e is a proper vector corresponding to a positive value and it is a longitudinal amplitude.

In what follows we suppose that:

iii) $\mathcal{F}(\pi, \theta, \alpha)$ is an isotropic function with respect to (π, α). From (77) we obtain

$$\bar{\mathcal{F}}(T, \theta, \bar{\alpha}, R^e) \equiv \mathcal{F}(T, \theta, \bar{\alpha}) = \mathcal{F}(\pi, \theta, \alpha) \qquad (85).$$

iv) $\mathcal{F}$ dependes only on the deviatoric part of π and α_j, or on T' and $\bar{\alpha}'_j$, and $\operatorname{tr} \partial_{T'} \mathcal{F} = 0$.

PROPOSITION 5.3. Under the conditions i)-iv) with isotropic line as elastic constitutive equation (73) the symmetric acoustic tensor (75) becomes:

$$Q(n)v = (\bar{\lambda}^e + \bar{\mu}^e)(n \cdot v)n + \bar{\mu}^e v -$$

$$- (4h(\lambda)(\bar{\mu}^e)^2 \nu (\partial_{T'} \mathcal{F} v \cdot n))/(1 + 2\nu\bar{\mu}^e |\partial_{T'} \mathcal{F}|^2)\, \partial_{T'} \mathcal{F} n \qquad (87)$$

and $Q(n)n \cdot n > 0$ for all $v \in \mathcal{V}$, if and only if;

$$(\bar{\lambda}^e + 2\bar{\mu}^e) > ((4h(\lambda)(\bar{\mu}^e)^2\nu)/(1 + 2\nu\bar{\mu}^e |\partial_{T'}\mathcal{F}|^2))|\partial_{T'}\mathcal{F}|^2 \tag{88}$$

Proof. From iii) we replace $\partial_T\bar{\mathcal{F}}$ by $\partial_T\mathcal{F}$ and $\bar{\mathcal{E}}_{ijke} = \bar{\lambda}^e\delta_{ij}\delta_{ke} + \bar{\mu}^e(\delta_{ik}\delta_{je} + \delta_{ie}\delta_{jk})$. Therefore (87) follows from (75). Here the hardening parameter γ is calculated from (76) in which tr $\partial_{T'}\mathcal{F} = 0$ and $\bar{\mathcal{E}}\partial_{T'}\mathcal{F}\cdot\partial_{T'}\mathcal{F} = 2\bar{\mu}^e|\partial_{T'}\mathcal{F}|^2$.

The condition (88) is obtained from (83) if we note that

$$\bar{\mathcal{E}}\cdot(n\otimes n\otimes n\otimes n) = \bar{\lambda}^e + 2\bar{\mu}^e, \quad \sup_{n\in\mathcal{V},\ |n|=1} |\bar{\mathcal{E}}\partial_{T'}\mathcal{F}n\cdot n|^2 =$$

$$= 4(\bar{\mu}^e)^2 \sup_{\substack{n\in\mathcal{V}\\ |n|=1}} |\partial_{T'}\mathcal{F}n\cdot n|^2 =$$

$=4(\bar{\mu}^e)^2|\partial_{T'}\mathcal{F}|^2$. Here we have used the symmetry of $\partial_{T'}\mathcal{F}$.

REMARK. If $\bar{\lambda}^e > 0$ then for $\nu > 0$ the condition (88) is satisfied.

THEOREM 5.1. Under the conditions of P.5.3:

1) A wave-normal n such that $\partial_{T'}\mathcal{F}n\cdot n \neq 0$ is a longitudinal amplitude if and only if n is a proper vector of $\partial_{T'}\mathcal{F}$.

2) The local speed of the longitudinal wave in the direction e_i is given by $U = \sqrt{Q(e_i)e_i\cdot e_i/\rho}$ where

$$Q(e_i)e_i\cdot e_i = (\bar{\lambda}^e + 2\bar{\mu}^e) - ((4\nu(\bar{\mu}^e)^2h(\lambda))/(1 + 2\nu\bar{\mu}^e|\partial_{T'}\mathcal{F}|^2))(\partial_{T'}\mathcal{F}e_i\cdot e_i)^2 \tag{89}$$

must be positive.

3) A wave travelling in a direction n, such that $\partial_{T'}\mathcal{F}n\cdot n = 0$ allows a longitudinal amplitude which is propagating with the local elastic longitudinal speed given by $U_L = \sqrt{(\bar{\lambda}^e + 2\bar{\mu}^e)/\rho}$.

Proof. Let n be a longitudinal amplitude. Then s-the amplitude vector of the wave is paralel with n, i.e. $Q(n)n = \lambda(n)n$. From (87) we get

$$Q(n)n = (\bar{\lambda}^e + 2\bar{\mu}^e)n - ((4h(\lambda)(\mu^e)^2\nu(\partial_{T'}\mathcal{F}n\cdot n))/(1 + 2\nu\bar{\mu}^e|\partial_{T'}\mathcal{F}|^2))\,\partial_{T'}\mathcal{F}n \tag{90}$$

If $\partial_{T'}\mathcal{F}n\cdot n \neq 0$ then Q(n)n is paralel with n if and only if $\partial_{T'}\mathcal{F}n$ is paralel with n, i.e. n is a proper vector of $\partial_{T'}\mathfrak{F} \in$ Sym.

Conversely, let a wave-normal n, with n a given proper vector for $\partial_{T'}\mathfrak{F}$. There is $s \in V$ such that $Q(n)s = \lambda(n)s$. From (87) we obtain that $Q(n)s = an + \bar{\mu}^e s$, with $a \in R$ generally non-zero. Comparing this expression with $Q(n)s = \lambda(n)s$ we obtain that s is paralel with n.

3) If n is such that $\partial_{T'}\mathfrak{F}\, n \cdot n = 0$ then (90) gives $Q(n)n = (\bar{\lambda}^e + 2\bar{\mu}^e)n$, i.e. all these waves allow a longitudinal amplitude.

T.5.2. Let n be a wave-normal n which allows a transverse amplitude

a_1) If n is a proper vector of $\partial_{T'}\mathfrak{F}\, n$ then the amplitude vector of the transverse wave corresponds to any orthogonal direction to n and the local speed of the wave is elastic, i.e. $U_s = \sqrt{\bar{\mu}^e/\rho}$.

a_2) If n is not a proper vector of $\partial_{T'}\mathfrak{F}$ then the amplitude vector is paralel with the normal to $(n, \partial_{T'}\mathfrak{F}\, n)$. The local speed is also elastic.

b) If $\partial_{T'}\mathfrak{F}\, n \cdot n = 0$ and n is not a proper vector of $\partial_{T'}\mathfrak{F}$ then s is paralel with $\partial_{T'}\mathfrak{F}\, n \neq 0$ and the local speed of this transverse wave is real if and only if

$$Q(n)s \cdot s) = \bar{\mu}^e - ((4h(\lambda)\nu(\bar{\mu}^e)^2 |\partial_{T'}\mathfrak{F}\, n|^2)/(1 + 2\nu\bar{\mu}^e |\partial_{T'}\mathfrak{F}|^2)) > 0 \qquad (91)$$

for all $n \in V$ such that $\partial_{T'}\mathfrak{F}\, n \cdot n = 0$.

Proof. A wave-normal n allows a transverse amplitude if and only if there exists $s \in V$, $s \cdot n = 0$ such that s is a proper vector for Q(n). From (87) we obtain

$$Q(n)s = \bar{\mu}^e - ((4h(\lambda)(\bar{\mu}^e)^2 \nu (\partial_{T'}\mathfrak{F}\, s \cdot n))/(1 + 2\nu\bar{\mu}^e |\partial_{T'}\mathfrak{F}|^2))\, \partial_{T'}\mathfrak{F}\, n \qquad (92).$$

It results that Q(n)s is paralel with s either if a) $\partial_{T'}\mathfrak{F}\, s \cdot n = 0$, with $n \cdot s = 0$, or b) $\partial_{T'}\mathfrak{F}\, s \cdot n \neq 0$ but $\partial_{T'}\mathfrak{F}\, n$ is paralel with s and $s \cdot n = 0$. In the case a) the local speed is elastic and the statements a_1) and a_2) follow at once since $\partial_{T'}\mathfrak{F} \in$ Sym. If n is not a proper vector for $\partial_{T'}\mathfrak{F}$ and $\partial_{T'}\mathfrak{F}\, n \cdot n = 0$, then any s paralel with $\partial_{T'}\mathfrak{F}\, n$ is an amplitude vector, and from (92) we obtain (91).

As an example (see Dafalias [5] and Loret [6], but they neglected the rate of elastic deformation in (98)) we consider an isotropic linear elastic constitutive equation (73) with the associated plastic flow

$$D^p = \nu \langle \lambda \rangle \partial_{T'} \mathcal{F} \equiv \langle \lambda \rangle \{\bar{B}\}_s \tag{93}$$

related to the plastic function $\mathcal{F}(T, \bar{\alpha}_1, \alpha_2)$, say for instence:

$$\mathcal{F}(T, \bar{\alpha}_1, \alpha_2) = (1/2k^2)(T' - \bar{\alpha}_1) \cdot (T' - \bar{\alpha}_1) - 1 \qquad (94),$$

isotropic with respect to T and $\bar{\alpha}_1$ (the deviatoric shift or back-stress tensor) and which depends on $\alpha_2 \in R$. The actual plastic spin

$$W^p = \langle \lambda \rangle \{\bar{B}\}_a, \text{ with } \{\bar{B}\}_a = \eta(\bar{\alpha}_1 T' - T'\bar{\alpha}_1) \tag{94}$$

for instance, where $\{\bar{B}\}_a$ is an isotropic antisymmetric valued function with respect to T' and $\bar{\alpha}_1$ and depends on $\alpha_2 \in R$. The plastic loading factor becomes

$$\lambda = \partial_T \mathcal{F} \cdot \overset{\circ}{T}, \text{ with } \overset{\circ}{T} = \dot{T} - \Omega^e T + T\Omega^e, \ \Omega^e = \dot{R}^e (R^e)^T \tag{95}$$

for any stress state such that $\mathcal{F}(T', \bar{\alpha}_1, \alpha_2) = 0$, since (85) holds. The evolution equations for the actual value of internal variables $\bar{\alpha}_1$, $\bar{\alpha}_2 = \alpha_2$ are

$$\overset{\circ}{\bar{\alpha}}_1 = \langle \lambda \rangle \bar{m}_1, \text{ with } \overset{\circ}{\bar{\alpha}}_1 = \dot{\bar{\alpha}}_1 - \Omega^e \bar{\alpha}_1 + \bar{\alpha}_1 \Omega^e \tag{96}$$

and

$$\dot{\bar{\alpha}}_2 = \langle \lambda \rangle \bar{m}_2$$

$\bar{m}_j$ (j = 1, 2) supposed also to be isotropic with respect to T' and $\bar{\alpha}_1$, dependent of $\bar{\alpha}_2$. The function $\bar{m}_1$ may be given by

$$\bar{m}_1 \equiv c(\bar{\alpha}_2)(T' - \bar{\alpha}_1) + d(\bar{\alpha}_1, \bar{\alpha}_2)\bar{\alpha}_1 \tag{97}$$

and $\dot{\alpha}_2$ is defined by $\dot{\alpha}_2 = \sqrt{(3/2)}(D^p \cdot D^p)^{1/2}$.

The scalar constitutive functions from (93)-(97) are given such as to satisfy the consistency condition $(8)_1$ on S(t).

From (32) and (14) we obtain the following relations:

$$\{L\}_s \equiv D = D^p + \overset{\circ}{\varepsilon}{}^e, \text{ with } \overset{\circ}{\varepsilon}{}^e = \dot{\varepsilon}^e - \Omega^e \bar{\varepsilon}^e + \bar{\varepsilon}^e \Omega^e \tag{98}$$

and

$$\{L\}_a \equiv W = \Omega^e + W^p, \; \Omega^e = \dot{R}^e (R^e)^T$$

in the case of small elastic deformations (see (68)-(71)). Here $L \equiv \dot{F}F^{-1}$ represents the velocity gradient.

We observe that the rates $\overset{\circ}{T}$, $\overset{\circ}{\bar{\alpha}}_1$, $\overset{\circ}{\bar{\varepsilon}}{}^e$ for the spin Ω^e are objective tensors since $T^* = QTQ^T$, $\bar{\alpha}_1^* = Q\bar{\alpha}_1 Q^T$, $F^{*e} = QF^e$, $F^{*p} = F^p$ (see [9] and [12]), where by * we denote the fields with reference to the motion $\chi^*(X,t) = x_0^*(t) + Q(t)(\chi(X,t) - x_0)$, with $Q \equiv Q(t) \in \text{Orth}$.

The symmetry of the acoustic tensor follows and all the results contained in T.5.1.. T.5.2 hold too.

CONCLUSIONS. We consider an elastoplastic body defined by:

- linear isotropic elastic constitutive equation,

- associated plastic flow low related to a plastic function isotropic with respect to T and $\bar{\alpha}$ -the actual internal variables (with $\mathrm{tr}\,\partial_T \mathcal{F} = 0$) when the elastic deformations are small and the elastic rotation great and when $\overset{\circ}{\bar{\varepsilon}}{}^e$ is compared with D^p.

Then

(1) all the longitudinal and transvers a.w. are propagating with local elastic speeds, i.e. $U = U_L$ or $U = U_S$, when the elastoplastic state corresponds to a unloading, or neutral process, or the elastoplastic state is inside the current yield surface,

(2) when the elastoplastic state corresponds to a loading process (i.e. $\lambda > 0$), there exist some transverse a.w. and some longitudinal a.w. (see T.5.1 and T.5.2) which are propagating with local speeds less than the corresponding elastic speeds.

We note that if the material is plastic incompressible then $[\mathrm{div}\, \dot{\bar{\varepsilon}}{}^e] \neq 0$ on the longitudinal a.w. and $[\mathrm{div}\, \dot{\bar{\varepsilon}}{}^e] = 0$ on the transvers a.w.

In our analyse we have essentialy used the existence of the elastic rate of deformation which is comparable with the plastic rate of deformation. If $\overset{\circ}{\ddot{\varepsilon}}{}^{e}$ (in the case of small deformation) is neglected with respect to D^p (this means that $\bar{\mu}^e \to +\infty$) then the local speeds of the longitudinal and transvers waves become ∞.

In his experiments concerning the propagation of the plastic waves in pre-stressed bars (see [17] and also [18] for general remarks about this problem) Bell obtained a local elastic speed along the longitudinal reloading waves.

In our theoretical consideration we can obtain the elastic longitudinal reloading waves if we consider the viscoplastic terms A and ℓ in the evolution equations (5).

So, a more realistic model based on (93)-(97) can be obtained if we consider the elastic rate of deformation in $(98)_1$ as well as the viscoplastic terms in evolution equations: (93) with (94) and (97).

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