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EVOLUTION PROBLEMS FOR A CLASS OF THERMO-VISCOPLASTIC MATERIALS

by

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EVOLUTION PROBLEMS FOR A CLASS OF THERMO - VISCOPLASTIC MATERIALS

by

Mircea Sofonea

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1. INTRODUCTION

Two initial and boundary value problems for materials with a constitutive equation of the form

$$\dot{\varepsilon} = A\dot{\sigma} + G(\sigma,\theta) \qquad (1.1)$$

are considered, in which ε is the small strain tensor, σ is the stress tensor and θ is the absolute temperature. Such type of equations are used in order to model the behaviour of real bodies like rubbers, metals, rocks and so on, for which the plastic rate of deformation depends also on the temperature.

In the isothermal case, (1.1) reduces to

$$\dot{\varepsilon} = A\dot{\sigma} + G(\sigma) \qquad (1.2)$$

and represents a semilinear rate-type constitutive equation. Various results and mechanical interpretations concerning constitutive equations of the form (1.2) and also more general constitutive equations in which $A=A(\sigma,\varepsilon)$, $G=G(\sigma,\varepsilon)$ may be found for instance in the papers of Cristescu and Suliciu[6],[7]. In particular cases, equation (1.2) reduces to some classical models used in viscoplasticity.

Existence and uniqueness results in the

study of models of the form (1.2) may be found for instance in the papers of Duvaut and Lions [8] Ch.5 and Suquet [14],[15]. For a more complicate problem concerning models of the form (1.1) in which θ is a hardening parameter , an existence and uniqueness result is given by Laborde [11]. Moreover, the existence and behaviour of the solution for quasistatic processis concerning models of the form (1.2) in which G depends both on σ and ε is studied in the paper of Ionescu and Sofonea [10].

A relative simple example of constitutive equations satisfying all the mathematical requirements of this paper may be obtained taking in (1.1)

$$G(\sigma, \theta) = \frac{1}{2\mu}(\sigma - P_{K(\theta)}\sigma)$$

$$K(\theta) = \{\sigma \mid |\sigma^D| \leq k(\theta)\}$$

where $\mu>0$ is a viscosity coefficient, $P_{K(\theta)}$ is the projector map on von Mises plasticity convex $K(\theta)$ defined by the Lipschitz yield function $\theta \to k(\theta)$.

We consider for the heat flux denoted by q a Cattaneo-type heat conduction law of the form

$$A\dot{q} + q = K\nabla\theta \tag{1.3}$$

and also the classical Fourier law given by

$$q = K\nabla\theta \tag{1.4}$$

Heat conduction constitutive equations of the form (1.3) were introduced by Cataneo [4] and were studied by several authors.

Some of them studied the structure of the constitutive equations, others were dealing with some initial and boundary value problems (for references see Cristescu and Suliciu [7] p.190).

The aim of this paper is to study two uncoupled thermo-mechanical problems governed by the constitutive equations (1.1), (1.3) (problem C) and (1.1), (1.4) (problem F). Thus, imposing some regularity and compatibility conditions on the input data and using monotonicity arguments in Hilbert spaces, the existence and uniqueness of the solution for the problems C and F is obtained (theorems 4.1, 4.2). The convergence of the solution of problem C to the solution of problem F when $A \to 0$ is studied (theorem 4.3) and the continuous dependence of the solutions with respect the input data is also given (theorems 4.4, 4.5).

2. STATEMENT OF THE PROBLEMS

Let Ω be a bounded domain in R^N $(N=1,2,3)$ with a smooth boundary $\partial\Omega=\Gamma$ and let $\Gamma_1, \tilde{\Gamma}_1$ be open subsets of Γ; we denote by $\Gamma_2=\Gamma\setminus\overline{\Gamma}_1, \tilde{\Gamma}_2=\Gamma\setminus\overline{\tilde{\Gamma}}_1$, ν the outword unit normal vector on Γ, and by S the set of second order symmetric tensors on R^N. Let T be a real positive constant. We consider now the following mixt problems:

PROBLEM C. Find the velocity function $v:[0,T]\times\Omega \to R^N$, the stress function $\sigma:[0,T]\times\Omega\to S$, the temperature function $\theta:[0,T]\times\Omega \to R$ and the heat flux function $q:[0,T]\times\Omega \to R^N$ such that

$$\left.\begin{aligned} &\rho\dot{v}=\operatorname{Div}\sigma + b && (2.1)\\ &A\dot{\sigma} + G(\sigma,\ \theta)=\varepsilon(v) && (2.2)\\ &\dot{\theta} = \operatorname{div} q + r && (2.3)\\ &A\dot{q} + q = K\nabla\theta && (2.4) \end{aligned}\right]\ \text{in } \Omega\times(0,T)$$

$$\begin{aligned} &v = U && \text{on } \tilde{\Gamma}_1\times(0,T) && (2.5)\\ &\sigma\nu = F && \text{on } \tilde{\Gamma}_2\times(0,T) && (2.6)\\ &\theta = \Theta && \text{on } \Gamma_1\times(0,T) && (2.7)\\ &q\nu=Q && \text{on } \Gamma_2\times(0,T) && (2.8) \end{aligned}$$

$$\left.\begin{aligned} &v(0)=v_o && (2.9)\\ &\sigma(0)=\sigma_o && (2.10)\\ &\theta(0)=\theta_o && (2.11)\\ &q(0)=q_o && (2.12) \end{aligned}\right]\ \text{in } \Omega$$

PROBLEM F. Find the velocity function $v:[0,T]\times\Omega\to R^N$, the stress function $\sigma:[0,T]\times\Omega\to S$, the temperature function $\theta:[0,T]\times\Omega\to R$ and the heat flux $q:[0,T]\times\Omega\to R^N$ such that

$$\left.\begin{aligned} &\rho\dot{v} = \operatorname{Div}\sigma + b && (2.13)\\ &A\dot{\sigma} + G(\sigma,\theta)=\varepsilon(v) && (2.14)\\ &\dot{\theta} = \operatorname{div} q + r && (2.15)\\ &q = K\nabla\theta && (2.16) \end{aligned}\right\}\ \text{in } \Omega\times(0,\ T)$$

$$\begin{aligned} &v=U && \text{on } \tilde{\Gamma}_1\times(0,\ T) && (2.17)\\ &\sigma\nu=F && \text{on } \tilde{\Gamma}_2\times(0,T) && (2.18)\\ &\theta=\Theta && \text{on } \Gamma_1\times(0,T) && (2.19)\\ &q\nu=Q && \text{on } \Gamma_2\times(0,T) && (2.20) \end{aligned}$$

$$\left.\begin{array}{ll} v(0)=v_o & (2.21) \\ \sigma(0)=\sigma_o & (2.22) \\ \theta(0)=\theta_o & (2.23) \end{array}\right] \text{ in } \Omega .$$

The problem C represents a uncoupled problem for thermo-elastic-visco-plastic materials of the form (2.2) where A is a forth order tensor which may depend on spacial coordinates and $G:\Omega x S x R \to S$ is a constitutive function. In (2.2) and everywhere in this paper $\dot{f}$ denotes the partial derivative of f with respect to time. (2.1) is the balance of momentum equation where the function $b:[0,T]x\Omega \to R^N$ is the body force function and $\rho > 0$ is the mass density. (2.3) is the heat conduction equation where $r:[0,T]x\Omega \to R$ is the volume heat-supply. (2.4) is a Cattaneo-type heat conduction constitutive equation where A and K are second-order tensors on R^N and finnaly, the functions U,F,Θ,Q are the given boundary data and the functions $v_o,\sigma_o,\theta_o,q_o$ are the given initial data.

The meaning of the problem F is the same, except the fact that the Cattaneo law (2.4) is replaced by the Fourier law (2.16) and hence the initial condition (2.12) is omitted.

3. NOTATIONS AND PRELIMINARIES

We denote by "." the inner product on the spaces R^N, S and by $|\bullet|$ the euclidian norms on thes spaces. The following notations are also used:

$$H = L^2(\Omega),$$

$$\tilde{H} = \{\nu = (\nu_i) | \nu_i \in H, \ i=\overline{1,N}\},$$

$$\mathcal{H} = \{\sigma = (\sigma_{ij}) | \sigma_{ij} = \sigma_{ji} \in H, \ i,j=\overline{1,N}\},$$

$$H_1 = \{\theta \in H | \nabla\theta \in \tilde{H}\},$$

$$\tilde{H}_1 = \{v \in \tilde{H} | \ \varepsilon(v) \in \mathcal{H}\},$$

$$\tilde{H}_2 = \{q \in \tilde{H} | \ \mathrm{div}\ q \in H\},$$

$$\mathcal{H}_2 = \{\sigma \in \mathcal{H} | \ \mathrm{Div} \in \tilde{H}\},$$

where ∇, ε, div and Div are the partialderivative operators of the first order defined by

$$\nabla\theta = (\nabla_i\theta), \quad \nabla_i\theta = \frac{\partial\theta}{\partial x_i}, \quad i = \overline{1,N}, \theta \in H,$$

$$\varepsilon(v) = (\varepsilon_{ij}(v)), \quad \varepsilon_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \quad i,j = \overline{1,N}, \ v \in \tilde{H},$$

$$\mathrm{div}\ q = \frac{\partial q_i}{\partial x_i}, \qquad q \in \tilde{H},$$

$$\mathrm{Div}\ \sigma = (D_i\sigma), \quad D_i\sigma = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad i = \overline{1,N}, \ \sigma \in \mathcal{H}$$

The spaces $H, \tilde{H}, \mathcal{H}, H_1, \tilde{H}_1, \tilde{H}_2, \mathcal{H}_2$ are Hilbert spaces with respect to the canonical inner products given by

$$(\theta, \varphi)_H = \int_\Omega \theta\varphi dx \tag{3.1}$$

$$(u, v)_{\tilde{H}} = \int_\Omega u . v \ dx \tag{3.2}$$

$$(\sigma, \tau)_{\mathcal{H}} = \int_\Omega \sigma . \tau \ dx \tag{3.3}$$

$$(\theta, \varphi)_{H_1} = (\theta, \varphi)_H + (\nabla\theta, \nabla\varphi)_{\tilde{H}} \tag{3.4}$$

$$(u, v)_{\tilde{H}_1} = (u, v)_{\tilde{H}} + (\varepsilon(u), \varepsilon(v))_{\mathcal{H}} \tag{3.5}$$

$$(q, p)_{\tilde{H}_2} = (q, p)_{\tilde{H}} + (\mathrm{div}\ q, \mathrm{div}\ p)_H \tag{3.6}$$

$$(\sigma, \tau)_{\mathcal{H}_2} = (\sigma, \tau)_{\mathcal{H}} + (\mathrm{Div}\ \sigma, \mathrm{Div}\ \tau)_{\tilde{H}} \tag{3.7}$$

The norms induced by (3.1)-(3.7) are denoted by $||\bullet||_H, ||\bullet||_{\tilde{H}}, \ldots, ||\bullet||_{\mathcal{H}_2}$ respectively. Also, for any real normed space X we denote by X' its strong dual, by $||\bullet||_X, ||\bullet||_{X'}$

the norms on X and X' respectively and by $\langle\, ,\rangle_{X',X}$ the canonical duality pairing between X' and X. If in addition X is a real Hilbert space and $A:X\to X$ is a continuous symmetric and positively definite linear operator, we denote by $\langle\, ,\rangle_{A,X}$ and $\|\bullet\|_{A,X}$ the energetical product and the energetical norm induced by A on X.

Let $H_\Gamma=H^{1/2}(\Gamma)$, $\tilde{H}_\Gamma=(H^{1/2}(\Gamma))^N$ and $\gamma_o:H_1\to H_\Gamma$, $\tilde{\gamma}_o:\tilde{H}_1\to\tilde{H}_\Gamma$ the trace maps. We introduce the following closed subspaces of H_1 and $\tilde{H}_1$

$$V=\{\theta\in H_1 \mid \gamma_o\theta=0 \quad \text{on } \Gamma_1\}$$

$$\tilde{V}=\{v\in\tilde{H}_1 \mid \tilde{\gamma}_o v=0 \quad \text{on } \tilde{\Gamma}_1\}$$

and we denote

$$V_\Gamma=\{\xi\in H_\Gamma \mid \xi=0 \quad \text{on } \Gamma_1\}$$

$$\tilde{V}_\Gamma=\{\tilde{\xi}\in\tilde{H}_\Gamma \mid \tilde{\xi}=0 \quad \text{on } \tilde{\Gamma}_1\}$$

For every $q\in\tilde{H}_2$ there exist $\gamma_1 q\in H'_\Gamma$ and $\tilde{\gamma}_1\sigma\in\tilde{H}'_\Gamma$ such that

$$\langle\gamma_1 q,\gamma_o\theta\rangle_{H'_\Gamma,H_\Gamma}=(q,\nabla\theta)_{\tilde{H}}+(\text{div } q,\theta)_H \quad \text{for all } \theta\in H_1 \quad (3.8)$$

$$\langle\tilde{\gamma}_1\sigma,\tilde{\gamma}_o v\rangle_{\tilde{H}'_\Gamma,\tilde{H}_\Gamma}=(\sigma,\varepsilon(v))_{\mathcal{H}}+(\text{Div}\sigma, v)_{\tilde{H}} \quad \text{for all } v\in\tilde{H}_1 \quad (3.9)$$

and $\gamma_1:\tilde{H}_2\to H'_\Gamma$, $\tilde{\gamma}_1:\mathcal{H}_2\to\tilde{H}'_\Gamma$ are linear and continuous operators. We mean by $q\nu_{|\Gamma_2}$ and $\sigma\nu_{|\tilde{\Gamma}_2}$ the elements of V'_Γ and $\tilde{V}'_\Gamma$ respectively which are the restrictions of $\gamma_1 q$ and $\tilde{\gamma}_1\sigma$ on V_Γ and $\tilde{V}_\Gamma$. We denote

$$\mathcal{V} = \{q \in H_2 \mid q\nu|\Gamma_2 = 0\}$$
$$\tilde{\mathcal{V}} = \{\sigma \in \mathcal{H}_2 \mid \sigma\nu|\tilde{\Gamma}_2 = 0\}$$

The spaces $\mathcal{V}$ and $\tilde{\mathcal{V}}$ are closed subspaces in $\tilde{H}_2$ and $\mathcal{H}_2$. We get by using (3.8), (3.9)

$$(q,\nabla\theta)_{\tilde{H}} + (\mathrm{div}\, q,\theta)_H = 0 \quad \text{for all } \theta\in V \text{ and } q\in\mathcal{V} \tag{3.10}$$

$$(\sigma,\ \varepsilon(v))_{\mathcal{H}} + (\mathrm{Div}\sigma,\ v)_{\tilde{H}} = 0 \quad \text{for all } v\in\tilde{V} \text{ and } \sigma\in\tilde{\mathcal{V}} \tag{3.11}$$

Finally, for every real Hilbert space X we use the classical notations $W^{k,p}(0,T,X)$ ($k\in N$, $1\le p\le\infty$) (see for instance Brezis [3] Appendice and Barbu [1], Ch. I) and we denote by $\|\bullet\|_{k,p,X}$ the norms on these spaces; for k=0 the short notation $\|\bullet\|_{p,X}$ is used instead of $\|\bullet\|_{0,p,X}$. Thus, by $\|x\|_{k,p,X}$ we mean $\|x\|_{W^{k,p}(0,T,X)}$ and by $\|x\|_{p,X}$ we mean $\|x\|_{L^p(0,T,X)}$.

4. STATEMENT OF MAIN RESULTS

The following hypotheses are made:

$\rho\in L^\infty(\Omega)$ and there exists $\alpha>0$ such that $\rho(x)\ge\alpha$ a.e. in Ω. (4.1)

$A : \Omega\times S\to S$ is a symmetric and positively definite bounded tensor i.e. (4.2)

(a) $A(x)\sigma.\tau=\sigma.A(x)\tau$ for all $\sigma,\tau\in S$ a.e. in Ω

(b) There exists $\beta>0$ such that $A(x)\tau.\tau\ge\beta|\tau|^2$ for all $\tau\in S$ a.e. in Ω

(c) $A_{ijkh}\in L^\infty(\Omega)$ for all $i,j,k,h=\overline{1,N}$

$G:\Omega\times S\times R\to S$ is a montone and Lipschitz measurable function i.e

$$\left.\begin{array}{ll}\text{(a)} & (G(x,\sigma_1,\theta)-G(x,\sigma_2,\theta))\cdot(\sigma_1-\sigma_2)\geq 0 \quad \text{for all } \sigma_1,\sigma_2\in S,\ \theta\in R, \text{ a.e. in } \Omega \\ \text{(b)} & \text{There exist } L_1,L_2>0 \text{ such that} \\ & |G(x,\sigma_1,\theta_1)-G(x,\sigma_2,\theta_2)|\leq L_1|\sigma_1-\sigma_2|+L_2|\theta_1-\theta_2| \\ & \text{for all } \sigma_1,\sigma_2\in S,\ \theta_1,\theta_2\in R \text{ a.e. in } \Omega \\ \text{(c)} & G(x,0,0)=0 \quad \text{a.e. in } \Omega \\ \text{(d)} & x\to G(x,\sigma,\theta):\Omega\to R \text{ is measurable with respect to the} \\ & \text{Lebesgue measure on } \Omega, \text{ for all } \sigma\in S,\ \theta\in R\end{array}\right\} \quad (4.3)$$

$K:\Omega\to S$ is a symmetric and positively definite bounded tensor i.e.

$$\left.\begin{array}{ll}\text{(a)} & K(x)q.p=q.K(x)p \text{ for all } q,p\in R^N, \text{ a.e. in } \Omega \\ \text{(b)} & \text{There exists } \gamma>0 \text{ such that } K(x)q.q\geq\gamma|q|^2 \\ & \text{for all } q\in R^N \text{ a.e. in } \Omega \\ \text{(c)} & K_{ij}\in L^\infty(\Omega) \text{ for all } i,j=\overline{1,N}\end{array}\right\} \quad (4.4)$$

$K^{-1}A:\Omega\to S$ is a symmetric and positively definite bounded tensor i.e.

$$\left.\begin{array}{ll}\text{(a)} & K^{-1}A(x)q.p=q.K^{-1}A(x)p \text{ for all } q,p\in R^N, \text{ a.e. in } \Omega \\ \text{(b)} & \text{There exists } \delta>0 \text{ such that } K^{-1}A(x)q.q\geq\delta|q|^2 \\ & \text{for all } q\in R^N \text{ a.e. in } \Omega \\ \text{(c)} & (K^{-1}A)_{ij}\in L^\infty(\Omega) \text{ for all } i,j=\overline{1,N}\end{array}\right\} \quad (4.5)$$

$$b\in W^{1,2}(0,T,\tilde{H}) \qquad (4.6)$$

$$r\in W^{1,2}(0,T,H) \qquad (4.7)$$

$$U\in W^{2,2}(0,T,\tilde{H}_\Gamma) \qquad (4.8)$$

$$F\in W^{2,2}(0,T,\tilde{V}'_\Gamma) \qquad (4.9)$$

$$\Theta \in W^{2,2}(0,T,H_\Gamma) \tag{4.10}$$

$$Q \in W^{2,2}(0,T,V'_\Gamma) \tag{4.11}$$

$$v_o \in \tilde{H}_1,\ \sigma_o \in H_2 \tag{4.12}$$

$$\theta_o \in H_1,\ q_o \in \tilde{H}_2 \tag{4.13}$$

$$\tilde{\gamma}_o(v_o)_{|\tilde{\Gamma}_1} = U(0)_{|\tilde{\Gamma}_1},\ \sigma_o \nu_{|\tilde{\Gamma}_2} = F(0) \tag{4.14}$$

$$\gamma_o(\theta_o)_{|\Gamma_1} = \Theta(0)_{|\Gamma_1},\ q_o \nu_{|\Gamma_1} = Q(0) \tag{4.15}$$

The main existence and uniqueness results of this paper are the following:

Theorem 4.1. Under the hypotheses (4.1)-(4.15) there exists a unique solution of the problem C such that

$$v \in W^{1,\infty}(0,T,\tilde{H}) \cap L^\infty(0,T,\tilde{H}_1) \tag{4.16}$$

$$\sigma \in W^{1,\infty}(0,T,H) \cap L^\infty(0,T,H_2) \tag{4.17}$$

$$\theta \in W^{1,\infty}(0,T,H) \cap L^\infty(0,T,H_1) \tag{4.18}$$

$$q \in W^{1,\infty}(0,T,\tilde{H}) \cap L^\infty(0,T,\tilde{H}_2) \tag{4.19}$$

Theorem 4.2. Let be $q_o = K\nabla\theta_o$; then, under the hypothesis (4.1)-(4.4), (4.6)-(4.15) there exists a unique solution of the problem F which verifies (4.16), (4.17) and such that

$$\theta \in W^{1,\infty}(0,T,H) \cap W^{1,2}(0,T,H_1) \tag{4.20}$$

$$q \in W^{1,2}(0,T,\tilde{H}) \cap L^\infty(0,T,\tilde{H}_2) \tag{4.21}$$

Further on, for every $\xi > 0$ we substitute in (2.4) A by ξA and denote by $(v_\xi, \sigma_\xi, \theta_\xi, q_\xi)$ the solution of problem C with the input data $b, r, U, F, \Theta, Q, v_o, \sigma_o, \theta_o, q_o$ and by (v, σ, θ, q) the solution of problem F with the same input data $b, r, U, F, \Theta, Q, v_o, \sigma_o, \theta_o$. We have the following result:

<u>Theorem 4.3.</u> Under the hypotheses of theorems 4.1 and 4.2 for $\xi \to 0$ we have

$$v_\xi \to v \quad \text{in } L^\infty(0,T,\tilde{H}) \tag{4.22}$$

$$\sigma_\xi \to \sigma \quad \text{in } L^\infty(0,T,\mathcal{H}) \tag{4.23}$$

$$\theta_\xi \to \theta \quad \text{in } L^\infty(0,T,H) \tag{4.24}$$

$$q_\xi \to q \quad \text{in } L^2(0,T,\tilde{H}). \tag{4.25}$$

The continuous dependence of the solutions with respect the input data is given by the following theorems:

<u>Theorem 4.4.</u> Assume that (4.1)-(4.5) hold and let $(v_i, \sigma_i, \theta_i, q_i)$ be the solutions of problem C for the data $b_i, r_i, U_i, F_i, \Theta_i, Q_i, v_{oi}, \sigma_{oi}, \theta_{oi}, q_{oi}$ $i=1,2$ satisfying (4.6)-(4.15). Then, there exist some constants $C_1>0$ and $C_2>0$ independent on input data such that

$$\|\theta_1-\theta_2\|_{\infty,H}+\|q_1-q_2\|_{\infty,\tilde{H}} \le C_1(\|\theta_{o1}-\theta_{o2}\|_H+\|q_{o1}-q_{o2}\|_{\tilde{H}}+ \\ +\|r_1-r_2\|_{2,H}+\|\Theta_1-\Theta_2\|_{1,\infty,H_\Gamma}+\|Q_1-Q_2\|_{1,\infty,V'_\Gamma}) \tag{4.26}$$

$$\|v_1-v_2\|_{\infty,\tilde{H}}+\|\sigma_1-\sigma_2\|_{\infty,\mathcal{H}}\le C_2(\|v_{o1}-v_{o2}\|_{\tilde{H}}+\|\sigma_{o1}-\sigma_{o2}\|_{\mathcal{H}}+ \\ +\|\theta_{o1}-\theta_{o2}\|_H+\|q_{o1}-q_{o2}\|_{\tilde{H}}+\|b_1-b_2\|_{2,\tilde{H}}+\|r_1-r_2\|_{2,H}+ \\ +\|\Theta_1-\Theta_2\|_{1,\infty,H_\Gamma}+\|Q_1-Q_2\|_{1,\infty,V'_\Gamma}+\|U_1-U_2\|_{1,\infty,\tilde{H}_\Gamma}+\|F_1-F_2\|_{1,\infty,\tilde{V}'_\Gamma}) \tag{4.27}$$

<u>Theorem 4.5.</u> Assume that (4.1)-(4.4) hold and let $(v_i, \sigma_i, \theta_i, q_i)$ be the solutions of problem F for the data $b_i, r_i, U_i, F_i, \Theta_i, Q_i,$ $v_{oi}, \sigma_{oi}, \theta_{oi}, q_{oi}=K\nabla\theta_{oi}$ $i=1,2$ satisfying

(4.6)-(4.15). Then, there exist some constants $C_3>0$ and $C_4>0$ independent on input data such that

$$\|\theta_1-\theta_2\|_{\infty,H}+\|q_1-q_2\|_{2,\tilde{H}}\leq C_3(\|\theta_{o1}-\theta_{o2}\|_{H_1}+\|r_1-r_2\|_{2,H}+ \tag{4.28}$$
$$+\|\Theta_1-\Theta_2\|_{1,\infty,H_\Gamma}+\|Q_1-Q_2\|_{1,\infty,V'_\Gamma})$$

$$\|v_1-v_2\|_{\infty,\tilde{H}}+\|\sigma_1-\sigma_2\|_{\infty,H}\leq C_4(\|v_{o1}-v_{o2}\|_{\tilde{H}}+\|\sigma_{o1}-\sigma_{o2}\|_{H}+$$
$$+\|\theta_{01}-\theta_{02}\|_{H_1}+\|b_1-b_2\|_{2,\tilde{H}}+\|r_1-r_2\|_{2,H}+\|\Theta_1-\Theta_2\|_{1,\infty,H_\Gamma}+ \tag{4.29}$$
$$+\|Q_1-Q_2\|_{1,\infty,V'_\Gamma}+\|U_1-U_2\|_{1,\infty,\tilde{H}_\Gamma}+\|F_1-F_2\|_{1,\infty,\tilde{V}'_\Gamma})$$

Remark 4.1. The hypotheses (4.4.b) and (4.5.a) in connection with the thermal conductivity tensor K and the relaxation times tensor A can be justified using thermodynamical arguments (see Suliciu [13] and Coleman, Fabrizio and Owen [5]).

Remark 4.2. The results stated in Theorems 4.1-4.5 hold with minnor adjustements also in the case when (4.6)-(4.11) are replaced by weaker regularity assumptions.

Remark 4.3. In theorems 4.4 and 4.5 as well as everywhere in this paper C, C_i ($i\in N$) represent strictely positive generic constants which may depend on $\Omega, \Gamma_1, \tilde{\Gamma}_1, \Gamma_2, \tilde{\Gamma}_2, \rho$, $A, K^{-1}, K^{-1}A$, G but do not depend on time and on input data.

5. PROOF OF THE RESULTS

In order to give the proofs of theorems 4.1-4.5 we need some preliminary results:

Lemma 5.1. Let X be a real Hilbert space, $D(B)\subset X$ a dense subspace of X and $A:X\to X$, $G:[0,T]\times X\to X$, $B:D(B)\subset X\to X$ operators with the following properties:

$$\left.\begin{array}{l} A \text{ is a continuous symmetric and positively} \\ \text{definite linear operator} \end{array}\right\} \quad (5.1)$$

G is a monotone and Lipschitz continuous operator i.e. (5.2)

(a) $\langle G(t,x_1)-G(t,x_2),\ x_1-x_2\rangle_X \geq 0$ for all $t\in[0,T]$ and $x_1,x_2\in X$

(b) there exists $L,\tilde{L}>0$ such that

$$\|G(t_1,x_1)-G(t_2,x_2)\|_X \leq L|t_1-t_2|+\tilde{L}\|x_1-x_2\|_X$$

for all $t_1,t_2\in[0,T]$ and $x_1,x_2\in X$

$$\left.\begin{array}{l} B \text{ is a linear (unbounded) operator such that} \\ B^*=-B \text{ where } B^*:D(B^*)=D(B)\subset X\to X \text{ is the adjoint} \\ \text{of } B. \end{array}\right\} \quad (5.3)$$

Then, for every $f\in W^{1,1}(0,T,X)$ and $x_o\in D(B)$ there exists a unique function $x\in W^{1,\infty}(0,T,X)$ such that

$$\left.\begin{array}{l} A\dot{x}(t)+G(t,x(t))=Bx(t)+f(t) \quad \text{a.e. in } [0,T] \\ x(0)=x_o. \end{array}\right\} \quad (5.4)$$

Proof. In view of (5.1) it is sufficient to consider the case $A=I_X$=indentity of X. Using classical results of monotonicity in Hilbert spaces (see for instance Pascali and Sburlan [12] p.113, Brezis [3] p.34) from (5.3) and (5.2) it results that for every $t\in[0,T]$ the sum $G(t)+B^*$:

$:D(B)\subset X\to X$ is a closed and maximal operator; moreover, for every $\lambda>0$, $x\in X$ and $t,s\in[0,T]$ we have

$$||(I_X+\lambda(G(t)+B^*))^{-1}x-(I_X+\lambda(G(s)+B^*))^{-1}x||\le L\lambda|t-s|$$

hence lemma 5.1 follows from theorem 4.1 of Barbu [1] p.164 (see also Kato [10]).

<u>Lemma 5.2.</u> Under the hypotheses (4.4), (4.5), (4.7), (4.10), (4.11), (4.13), (4.14), (4.15) there exists a unique couple of functions (θ,q) which verifies (2.3), (2.4), (2.7), (2.8), (2.11), (2.12), (4.18), (4.19).

<u>Proof</u>. Using the properties of the trace maps, from (4.10) and (4.11) we obtain the existence of the functions $\hat{\theta}\in W^{2,2}(0,T,H_1)$, $\hat{q}\in W^{2,2}(0,T,\tilde{H}_2)$ such that

$$\hat{\theta}(t)=\Theta(t) \quad \text{on} \quad \Gamma_1 x(0,T) \tag{5.5}$$

$$\hat{q}(t)\nu=Q(t) \quad \text{on} \quad \Gamma_2 x(0,T) \tag{5.6}$$

$$||\hat{\theta}||_{1,\infty,H_1}\le C_1||\Theta||_{1,\infty,H_\Gamma} \tag{5.7}$$

$$||\hat{q}||_{1,\infty,\tilde{H}_2}\le C_2||Q||_{1,\infty,V'_\Gamma} \tag{5.8}$$

Considering the functions defined by

$$g=r+\operatorname{div}\hat{q}-\dot{\hat{\theta}} \quad , \quad h=\nabla\hat{\theta}-K^{-1}A\dot{\hat{q}}-K^{-1}\hat{q} \tag{5.9}$$

$$\tilde{\theta}=\theta-\hat{\theta} \ , \qquad \tilde{q}=q-\hat{q} \tag{5.10}$$

$$\tilde{\theta}_o=\theta_o-\hat{\theta}(0), \qquad \tilde{q}_o=q_o-\hat{q}(0), \tag{5.11}$$

it is easily to see that (θ,q) is a solution for the problem (2.3), (2.4), (2.7), (2.8), (2.11), (2.12) iff $(\tilde{\theta},\tilde{q})$ is a solution for the following initial and homogenuous boundary value problem

$$\left.\begin{array}{l}\left.\begin{array}{l}\dot{\tilde{\theta}} = \operatorname{div}\tilde{q} + g \\ K^{-1}A\dot{\tilde{q}} + K^{-1}\tilde{q} = \nabla\tilde{\theta} + h\end{array}\right] \text{ in } \Omega \times (0,T) \\ \tilde{\theta} = 0 \quad \text{on } \Gamma_1 \times (0,T) \\ \tilde{q}\nu = 0 \quad \text{on } \Gamma_2 \times (0,T) \\ \left.\begin{array}{l}\tilde{\theta}(0) = \tilde{\theta}_o \\ \tilde{q}(0) = \tilde{q}_o\end{array}\right] \text{ in } \Omega.\end{array}\right] \qquad (5.12)$$

Using lemma 5.1 in the following functional framework

$$X = H \times \tilde{H}, \quad D(B) = V \times \mathcal{V} \quad B = \begin{pmatrix} 0 & \operatorname{div} \\ \nabla & 0 \end{pmatrix},$$

$$Ax = \begin{pmatrix} \theta \\ K^{-1}Aq \end{pmatrix}, \quad G(t,x) = \begin{pmatrix} 0 \\ K^{-1}q \end{pmatrix}, \quad \text{for all } x = (\theta, q) \in X \text{ and } t \in [0,T]$$

$$f = \begin{pmatrix} g \\ h \end{pmatrix}, \quad x_o = \begin{pmatrix} \tilde{\theta}_o \\ \tilde{q}_o \end{pmatrix}, \quad x = \begin{pmatrix} \tilde{\theta} \\ \tilde{q} \end{pmatrix}$$

it results the existence and uniqueness of the solution of (5.12) with the regularity $\tilde{\theta} \in W^{1,\infty}(0,T,H)$, $\tilde{q} \in W^{1,\infty}(0,T,\tilde{H})$; since from (5.12) we get $\operatorname{div}\tilde{q} \in L^\infty(0,T,H)$, $\nabla\tilde{\theta} \in L^\infty(0,T,\tilde{H})$ it result in fact $\tilde{\theta} \in W^{1,\infty}(0,T,H) \cap L^\infty(0,T,V)$, $\tilde{q} \in W^{1,\infty}(0,T,\tilde{H}) \cap L^\infty(0,T,\mathcal{V})$ and, using (5.9)-(5.12) we get the statement of lemma 5.2.

<u>Lemma 5.3.</u> Let be $q_o = K\nabla\theta_o$; then, under the hypotheses (4.4), (4.7), (4.10), (4.11), (4.13), (4.15) there exists a unique couple of functions (θ, q) which verifies (2.15), (2.16), (2.19), (2.20), (2.23), (4.20), (4.21).

<u>Proof.</u> Let h be the function defined by

$$h = K\nabla\tilde{\theta} - \hat{q} \tag{5.13}$$

where $\hat{q}$ satisfies (5.6), (5.8) and $\hat{\theta}$ satisfies (5.5),(5.6).

Using $(5.9)_1$, (5.10), $(5.11)_1$ it is easely to see that (θ,q) is a solution for the problem (2.15), (2.16), (2.19), (2.20), (2.23) iff $(\tilde{\theta},\tilde{q})$ satisfies the following initial and homogeneous boundary value problem

$$\left.\begin{array}{ll} \left.\begin{array}{l}\dot{\tilde{\theta}} = \operatorname{div}\tilde{q} + g \\ \tilde{q} = K\nabla\tilde{\theta} + h\end{array}\right] & \text{in } \Omega \times (0,T) \\ \tilde{\theta} = 0 & \text{on } \Gamma_1 \times (0,T) \\ \tilde{q}\nu = 0 & \text{on } \Gamma_2 \times (0,T) \\ \tilde{\theta}(0) = \tilde{\theta}_o & \text{in } \Omega. \end{array}\right] \tag{5.14}$$

Using (3.10) from (5.14) it results

$$\left.\begin{array}{ll} (\dot{\tilde{\theta}},\varphi)_H + (K\nabla\tilde{\theta},\nabla\varphi)_{\tilde{H}} = (g,\varphi)_H - (h,\nabla\varphi)_{\tilde{H}} & \text{for all} \\ \tilde{\theta}(0) = \tilde{\theta}_o \qquad \varphi\in V & \text{, a.e. on } [0,T] \end{array}\right] \tag{5.15}$$

Applying a classical results concerning parabolic equations (see for instance Barbu [2] p.124) we get the existence and uniqueness of the solution of (5.15) with the regularity $\tilde{\theta}\in W^{1,\infty}(0,T,H)\cap W^{1,2}(0,T,V)$. Taking $\tilde{q}=K\nabla\tilde{\theta}+h$ we get $\tilde{q}\in W^{1,2}(0,T,\tilde{H})\cap L^\infty(0,T,\mathcal{V})$ and $(\tilde{\theta},\tilde{q})$ is a solution for (5.14) and hence lemma 5.3 is proved.

<u>Proof of theorem 3.1</u>

It is sufficient to prove the existence and uniqueness of a couple of functions (v,σ) which satisfies (2.1), (2.2), (2.5), (2.6), (2.9), (2.10), (4.16), (4.17) where θ is the function given by lemma 5.2. Using the properties of of the trace maps, from (4.8), (4.9) we obtain the existence of the functions $\hat{v}\in W^{2,2}(0,T,\tilde{H}_1)$, $\hat{\sigma}\in W^{2,2}(0,T,H_2)$ such that

$$\hat{v}(t) = U(t) \qquad \text{on } \tilde{\Gamma}_1 \times (0,T) \tag{5.16}$$

$$\hat{\sigma}(t)\nu = F(t) \qquad \text{on } \tilde{\Gamma}_2 \times (0,T) \tag{5.17}$$

$$||\hat{v}||_{1,\infty,\tilde{H}_1} \leq C_1 ||U||_{1,\infty,\tilde{H}_\Gamma} \tag{5.18}$$

$$||\hat{\sigma}||_{1,\infty,\tilde{H}_2} \leq C_2 ||F||_{1,\infty,\tilde{V}'_\Gamma} \tag{5.19}$$

Considering the functions defined by

$$\tilde{g} = b + \mathrm{Div}\,\hat{\sigma} - \rho\dot{\hat{v}}, \qquad \tilde{h} = \varepsilon(\hat{v}) - A\dot{\hat{\sigma}} \tag{5.20}$$

$$\tilde{v} = v - \hat{v}, \qquad \tilde{\sigma} = \sigma - \hat{\sigma} \tag{5.21}$$

$$\tilde{v}_o = v_o - \hat{v}(0), \qquad \tilde{\sigma}_o = \sigma_o - \hat{\sigma}(0) \tag{5.22}$$

it is easy to see that in problem under consideration reduces to the following initial and homogeneous boundary value problem:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\rho\dot{\tilde{v}} = \mathrm{Div}\,\tilde{\sigma} + \tilde{g} \\
A\dot{\tilde{\sigma}} + G(\tilde{\sigma} + \hat{\sigma}, \theta) = \varepsilon(\tilde{v}) + \tilde{h}
\end{array}\right] \text{ in } \Omega\times(0,T) \\
\tilde{v} = 0 \qquad \text{on } \tilde{\Gamma}_1 \times (0, T) \\
\tilde{\sigma}\nu = 0 \qquad \text{on } \tilde{\Gamma}_2 \times (0,T) \\
\left.\begin{array}{l}
\tilde{v}(0) = \tilde{v}_o \\
\tilde{\sigma}(0) = \tilde{\sigma}_o
\end{array}\right] \text{ in } \Omega.
\end{array}\right] \tag{5.23}$$

Using lemma 5.1 in the following functional framework

$$X = \tilde{H} \times H, \quad D(B) = \tilde{V} \times \tilde{\mathcal{V}}, \quad B = \begin{pmatrix} 0 & \mathrm{Div} \\ \varepsilon & 0 \end{pmatrix}, \quad Ax = \begin{pmatrix} \rho v \\ A\sigma \end{pmatrix},$$

$$\mathcal{G}(t,x)=\begin{pmatrix} 0 \\ G(\sigma+\hat{\sigma}(t),\theta(t)) \end{pmatrix}, \text{ for all } x=(v,\sigma)\in X \text{ and } t\in[0,T],$$

$$f=\begin{pmatrix}\tilde{g}\\ \tilde{h}\end{pmatrix}, \quad x=\begin{pmatrix}\tilde{v}\\ \tilde{\sigma}\end{pmatrix}, \quad x_0=\begin{pmatrix}\tilde{v}_0\\ \tilde{\sigma}_0\end{pmatrix}$$

it results the existence and uniqueness of the solution of (5.23) with the regularity $\tilde{v}\in W^{1,\infty}(0,T,\tilde{H})$, $\tilde{\sigma}\in W^{1,\infty}(0,T,\mathcal{H})$; since from (5.23) we get Div $\tilde{\sigma}\in L^\infty(0,T,\tilde{H})$, $\varepsilon(\tilde{v})\in L^\infty(0,T,\mathcal{H})$ in fact $\tilde{v}\in W^{1,\infty}(0,T,\tilde{H})\cap L^\infty(0,T,\tilde{V})$, $\tilde{\sigma}\in W^{1,\infty}(0,T,\mathcal{H})\cap L^\infty(0,T,\tilde{\mathcal{V}})$ and , using (5.20)-(5.23) we get the statement of theorem 4.1.

Proof of Theorem 4.2

Is the same as proof of theorem 4.1, replacing only lemma 5.2 by lemma 5.3.

Remark 5.1. The dynamic elastic-visco-plastic problem (2.1), (2.2), (2.5), (2.6), (2.9), (2.10) was studied by several authors, using differents methods; so, in the case when G does not depend on θ and F does not depend on time, an existence and uniqueness result was given by Duvaut and Lions [8] ch.5, using the parabolic regularization method. In the case when G does not depend explicitely on θ but both $\mathcal{A}$ and G depend on time, an existence and uniqueness result was given by Suquet [14] using a time-discretization method. In the case when θ is interpreted as a hardening parameter, an existence and uniqueness result was given by Laborde [11] using a Galertin-type procedure.

<u>Proof of theorem 4.3</u>

From (2.3), (2.4), (2.15), (2.16) and using (3.10) we get:

$$\left.\begin{aligned}(\dot\theta_\xi(t)-\dot\theta(t),\ \theta_\xi(t)-\theta(t))_H+\xi(\dot q_\xi(t),\ q_\xi(t)-&\\ -q(t))_{K^{-1}A,\tilde H}+&\\ +\|q_\xi(t)-q(t)\|^2_{K^{-1},\tilde H}=0\quad \text{a.e. on } [0,T]&\end{aligned}\right]\quad (5.24)$$

Hence, it results

$$\frac{1}{2}\frac{d}{dt}(\|\theta_\xi(t)-\theta(t)\|^2_H+\xi\|q_\xi(t)-q(t)\|^2_{K^{-1}A,\tilde H})\le$$

$$\le -\xi(\dot q(t),q_\xi(t)-q(t))_{K^{-1}A,\tilde H}\le\xi\|q(t)\|_{K^{-1}A,\tilde H}\|q_\xi(t)-q(t)\|_{K^{-1}A,\tilde H}$$

a.e. on [0, T] and, using a Gronwall-type lemma (see BREZIS [3] p.157) we get

$$(\|\theta_\xi(s)-\theta(s)\|^2_H+\xi\|q_\xi(s)-q(s)\|^2_{K^{-1}A,\tilde H})^{1/2}\le \quad (5.25)$$

$$\le\sqrt{\xi}(\|q_o-K\nabla\theta_o\|_{K^{-1}A,\tilde H}+\int_0^s\|\dot q(t)\|_{K^{-1}A,\tilde H}dt)\ \text{for all } s\in[0,T]$$

From (5.25) it results (4.24) and also

$$\sqrt{\xi}\|q_\xi(s)-q(s)\|_{K^{-1}A,\tilde H}\to 0 \text{ when } \xi\to 0,\ \text{for all } s\in[0,T]. \quad (5.26)$$

Using again (5.24), (2.11), (2.12), (2.23) we get

$$\frac{1}{2}\|\theta_\xi(s)-\theta(s)\|^2_H+\frac{\xi}{2}\|q_\xi(s)-q(s)\|^2_{K^{-1}A}-\frac{\xi}{2}\|q_o-K\nabla\theta_o\|^2_{K^{-1}A}+$$

$$+\xi\int_0^s(\dot{q}(t),\ q_\xi(t)-q)_{K^{-1}A,\tilde{H}}dt+\int_0^s||q_\xi(t)-q(t)||^2_{K^{-1},\tilde{H}}=0$$

for all $s\in[0,T]$

So, it results

$$\int_0^s||q_\xi(t)-q(t)||^2_{K^{-1},\tilde{H}}dt \le \frac{\xi}{2}||q_0-K\nabla\theta_0||^2_{K^{-1}A,\tilde{H}}+$$

$$+\xi\int_0^s||\dot{q}(t)||_{K^{-1}A,\tilde{H}}||q_\xi(t)-q(t)||_{K^{-1}A,\tilde{H}}\,dt$$

and using (5.26) we get (4.25).

In the same way, from (2.1), (2.2), (2.13), (2.14) and using (3.11) we obtain

$$(\rho\dot{v}_\xi-\rho\dot{v}(t),v_\xi(t)-v(t))_{\tilde{H}}+(A\dot{\sigma}_\xi(t)-A\dot{\sigma}(t),\ \sigma_\xi(t)-\sigma(t))_{\mathcal{H}}+$$

$$+(G(\sigma_\xi(t),\theta_\xi(t))-G(\sigma(t),\theta(t)),\ \sigma_\xi(t)-\sigma(t))_{\mathcal{H}}=\text{a.e. on } [0,T]$$

Using again (4.1), (4.2), (4.3.b), (2.9), (2.10), (2.21), (2.22) by integration it results

$$||v_\xi(s)-v(s)||^2_{\tilde{H}}+||\sigma_\xi(s)-\sigma(s)||^2_{\mathcal{H}}\le C_1\int_0^s||\theta_\xi(t)-\theta(t)||^2_H dt+$$

$$+C_2\int_0^s||\sigma_\xi(t)-\sigma(t)||^2_{\mathcal{H}}dt \quad \text{for all } s\in[0,\ T]$$

and from Gronwall's lemma and (4.24) we get (4.22) and (4.23).

<u>Remark 5.2</u>. Similar techniques as in the proof of theorem 4.3 are used by Duvaut and Lions [8], Suquet [14], Ionescu and Sofonea [10].

Remark 5.3. In the homogeneous and isotropic case (2.4) and (2.16) reduces to the equations $\xi\dot{q}+q=k\nabla\theta$, $q=k\nabla\theta$ in which $\xi>0$ is a relaxation time and $k>0$ is the heat conductivity coefficient; in this particular case, theorem 4.3 shows that the solution of problem C converges in the sense given by (4.22)-(4.25) to the solution of problem F when $\xi\to 0$.

Proof of theorem 4.4

As we have seen in the proofs of theorem 4.1 and lemma 5.2 we have

$$v_i=\tilde{v}_i+\hat{v}_i,\qquad \sigma_i=\tilde{\sigma}_i+\hat{\sigma}_i \qquad i=1,2 \tag{5.27}$$

$$\theta_i=\tilde{\theta}_i+\hat{\theta}_i,\qquad q_i=\tilde{q}_i+\hat{q}_i \qquad i=1,2 \tag{5.28}$$

where $(\hat{v}_i,\hat{\sigma}_i)$ satisfies (5.16)-(5.19) for the data U_i,F_i, $(\hat{\theta}_i,\hat{q}_i)$ satisfies (5.5)-(5.8) for the data Θ_i, Q_i, $(\tilde{v}_i,\tilde{\sigma}_i)$ satisfies (5.23), (5.20), (5.22) for the data b_i,v_{oi},σ_{oi} and $(\tilde{\theta}_i,\tilde{q}_i)$ satisfies (5.12), (5.9), (5.11) for the data r_i,θ_{oi},q_{oi}.

Using a standard technique from (5.12), (4.4), (4.5) we get

$$\|\tilde{\theta}_1(t)-\tilde{\theta}_2(t)\|_H+\|\tilde{q}_1(t)-\tilde{q}_2(t)\|_{\tilde{H}}\le C(\|\tilde{\theta}_{o1}-\tilde{\theta}_{o2}\|+\|\tilde{q}_{o1}-\tilde{q}_{o2}\|_{\tilde{H}}+$$
$$+\|g_1-g_2\|_{2,H}+\|h_1-h_2\|_{2,\tilde{H}} \qquad \text{for all } t\in[0,T]$$

and from (5.9), (5.11), (5.7), (5.8) it results

$$\|\tilde{\theta}_1-\tilde{\theta}_2\|_{\infty,H}+\|\tilde{q}_1-\tilde{q}_2\|_{\infty,\tilde{H}}\le C(\|\theta_{o1}-\theta_{o2}\|_H+\|q_{o1}-q_{o2}\|_{\tilde{H}}+$$

$$+||r_1-r_2||_{2,H}+||\Theta_1-\Theta_2||_{1,\infty;H_\Gamma}+||Q_1-Q_2||_{1,\infty,V'_\Gamma}).$$

Using again (5.28), (5.7), (5.8) we get (4.26).

In the same way from (5.23), (4.1), (4.2) and (4.3b) we get

$$||\tilde{v}_1(t)-\tilde{v}_2(t)||_{\tilde{H}}+||\tilde{\sigma}_1(t)-\tilde{\sigma}_2(t)||_{H}\leq C(||\tilde{v}_{o1}-\tilde{v}_{o2}||_{\tilde{H}}+||\tilde{\sigma}_{o1}-\tilde{\sigma}_{o2}||_{H}+$$

$$+||\theta_1-\theta_2||_{2,H}+||\tilde{g}_1-\tilde{g}_2||_{2,\tilde{H}}+||\tilde{h}_1-\tilde{h}_2||_{2,H}+||\hat{\sigma}_1-\hat{\sigma}_2||_{2,H}$$

for all $t\in[0,T]$

and from (5.20), (5.22), (5.18), (5.19) it results

$$||\tilde{v}_1-\tilde{v}_2||_{\infty,\tilde{H}}+||\tilde{\sigma}_1-\tilde{\sigma}_2||_{\infty,H}\leq C(||v_{o1}-v_{o2}||_{\tilde{H}}+||\sigma_{o1}-\sigma_{o2}||_{H}+$$

$$+||\theta_1-\theta_2||_{2,H}+||b_1-b_2||_{2,\tilde{H}}+||U_1-U_2||_{1,\infty,\tilde{H}_\Gamma}+||F_1-F_2||_{1,\infty,\tilde{V}'_\Gamma}).$$

Using again (5.27), (5.18), (5.19) and (4.26) we get (4.27).

<u>Proof of theorem 4.5</u>

As we have seen in the proof of lemma 5.3 we have (5.28) where $(\hat{\theta}_i, \hat{q}_i)$ satisfies (5.5)-(5.8) for the data Θ_i, Q_i and $(\tilde{\theta}_i, \tilde{q}_i)$ satisfies (5.14), $(5.9)_1$, (5.13), (5.11) for the data r_i, θ_{oi}. Using (5.14) and (4.4) it results

$$||\tilde{\theta}_1(s)-\tilde{\theta}_2(s)||^2_H+\int_0^s||\tilde{q}_1(t)-\tilde{q}_2(t)||^2_{\tilde{H}}\leq C(||\tilde{\theta}_{o1}-\tilde{\theta}_{o2}||^2_H+||g_1-g_2||^2_{2,H}+$$

$$+\int_0^s||\tilde{\theta}_1(t)-\tilde{\theta}_2(t)||^2_H dt+\int_0^s||h_1(t)-h_2(t)||_{\tilde{H}}||\tilde{q}_1(t)-\tilde{q}_2(t)||_{\tilde{H}}dt)$$

for all $s\in[0,T]$

and hence

$$||\tilde{\theta}_1(s)-\tilde{\theta}_2(s)||_H^2 + \int_0^s ||\tilde{q}_1(t)-\tilde{q}_2(t)||_{\tilde{H}}^2 \le C(||\tilde{\theta}_{o1}-\tilde{\theta}_{o2}||_H^2 +$$

$$+||g_1-g_2||_{2,H}^2+||h_1-h_2||_{2,\tilde{H}}^2+ \int_0^s ||\tilde{\theta}_1(t)-\tilde{\theta}_2(t)||_H^2 dt \text{ for all } s\in[0,T].$$

Using Gronwall's lemma, after some manipulations we get

$$||\tilde{\theta}_1-\tilde{\theta}_2||_{\infty,H}+||\tilde{q}_1-\tilde{q}_2||_{2,\tilde{H}} \le C(||\tilde{\theta}_{o1}-\tilde{\theta}_{o2}||_H+||g_1-g_2||_{2,H}+||h_1-h_2||_{2,\tilde{H}})$$

From $(5.9)_1$, (5.13), $(5.11)_1$, (5.7), (5.8) it results

$$||\tilde{\theta}_1-\tilde{\theta}_2||_{\infty,H}+||\tilde{q}_1-\tilde{q}_2||_{2,\tilde{H}} \le C(||\theta_{o1}-\theta_{o2}||_H+||r_1-r_2||_{2,H}+$$
$$+||\Theta_1-\Theta_2||_{1,\infty,H_\Gamma}+||Q_1-Q_2||_{1,\infty,V_\Gamma'})$$

and using again (5.28), (5.7) and (5.8) we get (4.28). The estimation (4.29) results in the same way as (4.27) using (4.28) instead of (4.26).

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