A MATHEMATICAL MODEL OF THE BINARY SYSTEMS DRAWING

by

M.SANDRU*) and G.CAMENSCHI*)

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*) Department of Mechanics, University of Bucharest, Academiei 14, 70109 Bucharest, Romania.

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N. ȘANDRU and G. CAMENSCHI
Department of Mechanics, University of Bucharest

1. INTRODUCTION

The present technological materials must have the capacity to support strains, temperatures and other external influences and to ensure a high electrical conductivity in certain circumstances.

Such requerements are satisfied in some cases by composite materials, particularly by bimetals. One of the causes limitting bimetallic material forming is an insufficient knowlege of the working out process. In thin bars and bimetallic wire manufacturing the final operation is, as known, the material drawing through a conical converging die.

Unlike in the process of monometals drawing the existence of two materials having different mechanical properties provides specific characteristics to this process, thus the core may break or/and the external cover defoliates. This enforces the ensurance of an optimal course for the process, the diminushing of rejects and of scarce materials consumption, as well as, the increasing of the finite product quality [8].

This paper concerns a mathematical model of the bimetallic wire and bar drawing, using viscoplastic constitutive equations and integrating the governing system by means of the perturbation method. The drawing force is determined as a function of certain parameters of the process, proving by some numerical examples the process optimization possibility. The mathematical method used for the integration of the governing equations is the one we have used in our papers [1 - 7] for various metal processing procedures.

A binary system is a material continuum, occupying a region $\mathcal{D} = \mathcal{D}_{(1)} \cup \mathcal{S} \cup \mathcal{D}_{(2)}$, possessing a discontinuous homogeneity along a separating surface $\mathcal{S}$ between $\mathcal{D}_{(1)}$ and $\mathcal{D}_{(2)}$.

On the surface $\mathcal{S}$ the constants of the material are discontinuous.

In case the domain $\mathcal{D}$ and the surface $\mathcal{S}$ have an axis of symmetry such a binary system represents the theoretical model of a bimetallic bar or wire.

The viscoplastic deformation, described by Bingham type constitutive equations of the bimetal , takes place in the domain

$$\mathcal{D} = \{ (r, \theta, \varphi) / r \in (r_{(i)2}(\theta), r_{(i)1}(\theta), \theta \in [0, \alpha), \varphi \in [0, 2\pi) \},$$

called the deformation zone, where $r = r_{(i)j}(\theta)$, $i, j = 1, 2$, are the equations of the unknown discontinuity surfaces of the velocities and the stress $\mathcal{S}_{(i)j}$, $i, j = 1, 2$ at the entrence and exit of the material from the dye; r, θ, φ are the spherical coordinates with the origin in the top of the dye cone. The regions $\mathcal{D}_{(1)}$ and $\mathcal{D}_{(2)}$ are $\mathcal{D}_{(1)} = \{ (r, \theta, \varphi) / r \in (r_{(1)2}(\theta), r_{(1)1}(\theta)), \theta \in [0, \theta_s(r)), \varphi \in [0, 2\pi) \}$ and $\mathcal{D}_{(2)} = \{ (r, \theta, \varphi) / r \in (r_{(2)2}(\theta), r_{(2)1}(\theta)), \theta \in (\theta_s(r), \alpha), \varphi \in [0, 2\pi) \}$ where $\theta = \theta_s(r)$ is the separation surface equation, apriori unknown, denoted by $\mathcal{S}$, of the two material components.

Up to the entrence into the deformation zone and after the exit of this zone, the material has a rigid motion characterized by the following geometrical dimensions: R_3 , R_4 are the radius at the entrance and exit, respectively, of the core, R_1 and R_2 which are the radiud at the entrance and exit of the binary system and v_1 and v_2 which are the constant velocities of the rigid material at the entrence and exit of the die.

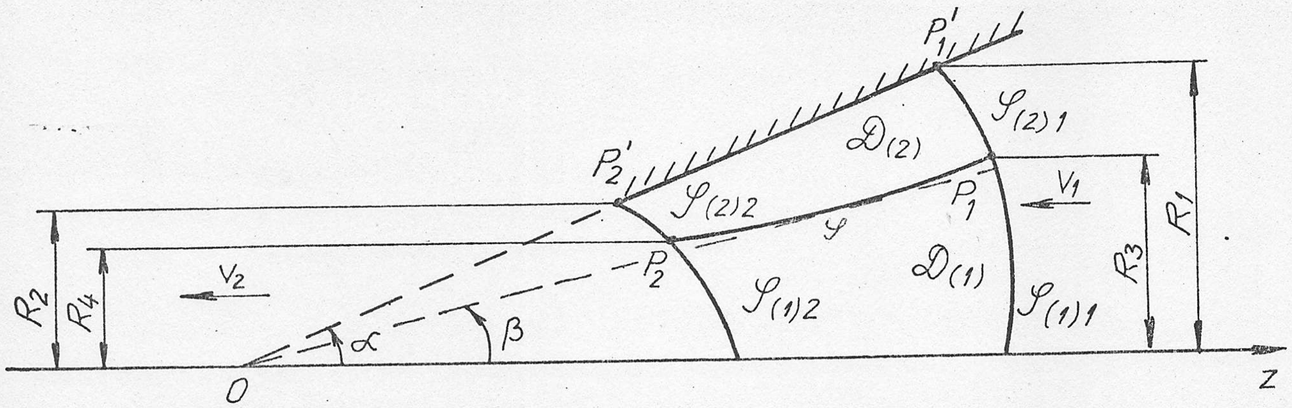

Fig. 1. Geometry of the binary system drawing

2. EQUATIONS OF THE PROBLEM

The governing equations in the $\mathcal{D}_{(i)}$, $i=1,2$ domains are:

- the Bingham type constitutive equations

$$\mathbb{t}_{(i)} = -p_{(i)}\mathbb{1} + \left(2\eta_{(i)} + \frac{k_{(i)}}{\sqrt{\mathrm{II}_{\mathbb{d}_{(i)}}}}\right)\mathbb{d}_{(i)}, \quad \mathrm{II}_{\mathbb{t}'_{(i)}} > k^2_{(i)}, \tag{2.1}$$

- the Cauchy's equations(in the absence of the body forces)

$$\rho_{(i)} \frac{d\vec{v}_{(i)}}{dt} = \mathrm{Div}\,\mathbb{t}_{(i)}, \tag{2.2}$$

- the continuity equation for incompressible material

$$\mathrm{I}_{\mathbb{d}_{(i)}} = 0, \tag{2.3}$$

In (2.1)-(2.3) $\vec{v}_{(i)}$ is the velocity vector, $\mathbb{t}_{(i)}$ is the Cauchy stress tensor, $\mathbb{d}_{(i)}$ is the rate of deformation tensor, $p_{(i)}$ is the pressure, $\mathrm{I}_{\mathbb{d}_{(i)}}$ and $\mathrm{II}_{\mathbb{d}_{(i)}}$ are the first two invarignts of the rate of deformation tensor, $\mathbb{t}'_{(i)}$ is the stress deviator, $\rho_{(i)}$ is the density, $\eta_{(i)}$ and $k_{(i)}$ are material constants.

Equation (2.3) leads easily to the following relations

$$v_1 R_3^2 = v_2 R_4^2, \quad v_1 R_1^2 = v_2 R_2^2. \tag{2.4}$$

By writing (2.1)-(2.3) in spherical coordinates and turning them into non-dimensional form by means of the relations

$$\begin{gathered} r = r^{\circ} R_2, \quad v_{(i)r} = v^{\circ}_{(i)r} v_2, \\ v_{(i)\theta} = v^{\circ}_{(i)\theta} v_2, \quad p_{(i)} = p^{\circ}_{(i)} \frac{\eta_{(i)} v_2}{R_2}, \quad i=1,2 \end{gathered} \tag{2.5}$$

the following dimensionless combinations are put into evidence

$$Bg_{(i)} = \frac{k_{(i)} R_2}{\eta_{(i)} v_2}, \quad Re_{(i)} = \frac{\rho_{(i)} v_2 R_2}{\eta_{(i)}}, \quad i=1,2 \tag{2.6}$$

that is the Bingham and Reynolds numbers.

We shall suppose that

$$Re_{(i)} << 1, \quad i=1,2 \tag{2.7}$$

therefore neglecting the inertial terms in (2.2).

Equation (2.3) in spherical coordinates has the following form

$$\frac{\partial v_{(i)r}}{\partial r} + \frac{1}{r}\frac{\partial v_{(i)\theta}}{\partial \theta} + \frac{2 v_{(i)r}}{r} + \frac{v_{(i)\theta}\,\mathrm{cotg}\,\theta}{r} = 0, \tag{2.8}$$

which allows the introduction of the stream functions

$$\Psi_{(i)}(r,\theta) = R_2^2 v_2 \overset{\circ}{\Psi}_{(i)}(r^\circ,\theta), \; i=1,2$$

in the domains $\mathcal{D}_{(i)}$ by means of the relations

$$\begin{aligned} \overset{\circ}{v}_{(i)r} &= -\frac{1}{r^{\circ 2}\sin\theta}\frac{\partial \overset{\circ}{\Psi}_{(i)}}{\partial\theta}, \\ \overset{\circ}{v}_{(i)\theta} &= \frac{1}{r^\circ \sin\theta}\frac{\partial \overset{\circ}{\Psi}_{(i)}}{\partial r^\circ}, \end{aligned} \qquad i=1,2. \tag{2.9}$$

By assuming that $Bg_{(i)} < 1$ and following the method used in [1 - 7] we shall develop the unknown functions $\overset{\circ}{\Psi}_{(i)}$, $\overset{\circ}{p}_{(i)}$ in power series with respect to the Bingham's number

$$\begin{aligned} \overset{\circ}{\Psi}_{(i)}(r^\circ,\theta) &= \sum_{j=0}^{\infty} Bg_{(i)}^{j}\, \overset{\circ}{\Psi}_{(i)j}(r^\circ,\theta), \\ \overset{\circ}{p}_{(i)}(r^\circ,\theta) &= \sum_{j=0}^{\infty} Bg_{(i)}^{j}\, \overset{\circ}{p}_{(i)j}(r^\circ,\theta), \end{aligned} \tag{2.10}$$

where $\overset{\circ}{\Psi}_{(i)0} = \overset{\circ}{\Psi}_{(i)0}(\theta)$ and $\overset{\circ}{\Psi}_{(i)1} = r^{\circ 3}\varphi_{(i)}(\theta)$.

By substituting the developments (2.10) in the Cauchy's equations, in the constitutive equations written in spherical coordinates and equating the terms having the same degree in $Bg_{(i)}$, one obtains the system of equations for $\overset{\circ}{\Psi}_{(i)0}, \overset{\circ}{\Psi}_{(i)1}, \ldots\; i=1,2$ i.e. the approximations of the stream functions and $\overset{\circ}{p}_{(i)0}, \overset{\circ}{p}_{(i)1}, \ldots\; i=1,2$ i.e. the pressures.

Turning back to dimensional variables and functions one gets

$$\begin{aligned} \Psi_{(i)}(r,\theta) &= R_2^2 v_2 \overset{\circ}{\Psi}_{(i)0}(\theta) + \frac{k_{(i)}}{\eta_{(i)}} r^3 \varphi_{(i)}(\theta) + O(Bg_{(i)}^2), \\ v_{(i)r}(r,\theta) &= -\frac{R_2^2 v_2}{r^2} u_{(i)}(\theta) - \frac{k_{(i)}}{\eta_{(i)}} r v_{(i)}(\theta) + O(Bg_{(i)}^2), \\ v_{(i)\theta}(r,\theta) &= 3\frac{k_{(i)}}{\eta_{(i)}} r \frac{\varphi_{(i)}(\theta)}{\sin\theta} + O(Bg_{(i)}^2), \quad i=1,2 \end{aligned} \tag{2.11}$$

where the functions

$$\begin{aligned} u_{(i)}(\theta) &= \frac{1}{\sin\theta}\frac{d\overset{\circ}{\Psi}_{(i)0}}{d\theta}, \\ v_{(i)}(\theta) &= \frac{1}{\sin\theta}\frac{d\varphi_{(i)}}{d\theta}, \quad i=1,2 \end{aligned} \tag{2.12}$$

may be written in the form

$$u_{(i)}(\theta) = a_{(i)} + b_{(i)}p(\theta) + d_{(i)}q(\theta),$$

$$v_{(i)}(\theta) = \frac{A_{(i)}}{6} + B_{(i)}p(\theta) + D_{(i)}q(\theta) + \tag{2.13}$$
$$+ K_{(i)1}(\theta)p(\theta) + K_{(i)2}(\theta)q(\theta), \quad i=1,2$$

with $d_{(1)} = D_{(1)} = 0$, where we used the notations

$$p(\theta) = \frac{1}{3} + \cos 2\theta,$$

$$q(\theta) = \left(\frac{1}{3} + \cos 2\theta\right)\ln\left(tg\frac{\theta}{2}\right) - (1 - 3\cos\theta)(1+\cos\theta). \tag{2.14}$$

The functions $K_{(i)1}(\theta)$ and $K_{(i)2}(\theta)$ satisfy the equations

$$\frac{dK_{(i)1}}{d\theta} = -\frac{9}{16} f_{(i)}(\theta)\sin\theta \cdot q(\theta),$$

$$\frac{dK_{(i)2}}{d\theta} = \frac{9}{16} f_{(i)}(\theta)\sin\theta \cdot p(\theta), \quad i=1,2 \tag{2.15}$$

where

$$f_{(i)}(\theta) = \frac{12u_{(i)} - u'_{(i)}\cot g\theta}{2\sqrt{3u_{(i)}^2 + \frac{u'^2_{(i)}}{4}}} -$$
$$- \frac{d}{d\theta}\left(\frac{u'_{(i)}}{2\sqrt{3u_{(i)}^2 + u'^2_{(i)}/4}}\right), \quad i=1,2 \tag{2.16}$$

The physical components of the stress tensor are given by

$$t_{(i)rr} = -\frac{2\eta_{(i)}v_2R_2^2}{r^3}\left[a_{(i)} - 3u_{(i)}(\theta)\right] -$$
$$- \frac{\eta_{(i)}v_2}{R_2}c_{(i)} + k_{(i)}\Big[A_{(i)}\ln\frac{r}{R_2} - h_{(i)}(\theta) +$$
$$+ \frac{2u_{(i)}(\theta)}{\sqrt{3u_{(i)}^2 + \frac{u'^2_{(i)}}{4}}} - 2v_{(i)}(\theta)\Big] + O(Bg^2_{(i)}),$$

$$t_{(i)\theta\theta} = -\frac{2\eta_{(i)}v_2R_2^2}{r^3}a_{(i)} - \frac{\eta_{(i)}v_2}{R_2}c_{(i)} +$$
$$+ k_{(i)}\Big[A_{(i)}\ln\frac{r}{R_2} - h_{(i)}(\theta) + 4v_{(i)}(\theta) - \tag{2.17}$$
$$- 6\cot g\theta \cdot \frac{\varphi_{(i)}(\theta)}{\sin\theta} - \frac{u_{(i)}}{\sqrt{3u_{(i)}^2 + \frac{u'^2_{(i)}}{4}}}\Big] +$$
$$+ O(Bg^2_{(i)}),$$

$$t_{(i)\varphi\varphi} = -\frac{2\eta_{(i)} v_2 R_2^2}{r^3} a_{(i)} - \frac{\eta_{(i)} v_2}{R_2} c_{(i)} + k_{(i)} \Big[A_{(i)} \ln \frac{r}{R_2} -$$

$$- h_{(i)}(\theta) - 2 v_{(i)}(\theta) + 6 \cot g\,\theta \cdot \frac{\varphi_{(i)}(\theta)}{\sin\theta} -$$

$$- \frac{u_{(i)}}{\sqrt{3u_{(i)}^2 + \frac{u_{(i)}'^2}{4}}} \Big] + O(Bg_{(i)}^2),$$

$$t_{(i)r\theta} = -\frac{\eta_{(i)} v_2 R_2^2}{r^3} u'_{(i)}(\theta) - k_{(i)} \Big[v'_{(i)}(\theta) +$$

$$+ \frac{u'_{(i)}}{2\sqrt{3u_{(i)}^2 + \frac{u_{(i)}'^2}{4}}} \Big] + O(Bg_{(i)}^2),$$

$$t_{(i)\theta\varphi} = t_{(i)r\varphi} = 0, \quad i = 1,2$$

where

$$h'_{(i)}(\theta) = v'_{(i)}(\theta) + \frac{6\varphi_{(i)}(\theta)}{\sin\theta} - \tag{2.18}$$

$$- \frac{d}{d\theta}\left(\frac{u_{(i)}(\theta)}{\sqrt{3u_{(i)}^2 + \frac{u_{(i)}'^2}{4}}} \right) - \frac{3u'_{(i)}(\theta)}{2\sqrt{3u_{(i)}^2 + \frac{u_{(i)}'^2}{4}}},$$

3. THE BOUNDARY OF THE BINARY SYSTEM DEFORMATION ZONE

On the surfaces $\mathcal{S}_{(i)j}$, $i, j = 1,2$, considered as discontinuity surfaces for the velocity and stress fields, according to the previous assumptions, the following relations are satisfied

$$[v_{(i)n}] = 0,$$
$$[t_{(i)kl} n_k] = 0, \tag{3.1}$$

where $\vec{n}$ is the normal unit vector to the surface.

If $r = r_{(i)j}(\theta)$, $i,j = 1,2$ with $\theta \in [0, \theta_s(r)]$ for $i = 1$ and $\theta \in [\theta_s(r), \alpha]$ for $i = 2$, are the equations of the surfaces $\mathcal{S}_{(i)j}$ then by neglecting the terms $O(Bg_{(i)}^2)$ in $(3.1)_1$ we get

$$R_2^2 v_2 \Psi^0_{(i)0}(\theta) + \frac{k_{(i)}}{r} r^3_{(i)j}(\theta) \cdot \varphi_{(i)}(\theta) - \tag{3.2}$$
$$- \frac{v_j}{2} r^2_{(i)j}(\theta) \sin^2\theta = C_{(i)j}, \quad i,j = 1,2.$$

If we require $\mathcal{S}_{(2)j}$ to pass through P'_j, $j = 1,2$ then results

$$C_{(2)j} = R_2^2 v_2 \Psi^0_{(2)0}(\alpha) - \frac{v_j}{2} R_j^2, \quad j = 1,2. \tag{3.3}$$

Therefore the equations for $\mathcal{S}_{(2)j}$ are given by

$$R_2^2 v_2 [\Psi^{\circ}_{(2)o}(\theta) - \Psi^{\circ}_{(2)o}(\alpha)] + \frac{k_{(2)}}{\eta_{(2)}} r^3_{(2)j} \varphi_{(2)}(\theta) - \\ - \frac{v_j}{2}[r^2_{(2)j} \sin^2\theta - R_j^2] = 0, \quad j = 1,2. \tag{3.4}$$

By assuming that the equation of the surface $\mathcal{S}$ has the form

$$\theta = \beta + Bg_{(1)} \cdot \beta_1 \frac{r^3}{R_2^3} + O(Bg^2_{(1)}), \tag{3.5}$$

where β and β_1 are constants, the components of the normal unit vector to this surface will be

$$n_r = \frac{r\theta_s'(r)}{\sqrt{1+r^2\theta_s'^2(r)}} = 3Bg_{(1)} \cdot \beta_1 \frac{r^3}{R_2^3} + O(Bg^2_{(1)}), \\ n_\theta = -\frac{1}{\sqrt{1+r^2\theta_s'^2(r)}} = -1 + O(Bg^2_{(1)}). \tag{3.6}$$

The continuity conditions of the velocities and stresses on the surface $\mathcal{S}$ are

$$v_{(1)r}/_{\mathcal{S}} = v_{(2)r}/_{\mathcal{S}}, \quad v_{(1)\theta}/_{\mathcal{S}} = v_{(2)\theta}/_{\mathcal{S}}, \tag{3.7}$$

and

$$t_{(1)rr} n_r + t_{(1)r\theta} n_\theta /_{\mathcal{S}} = t_{(2)rr} n_r + t_{(2)r\theta} n_\theta /_{\mathcal{S}}, \\ t_{(1)r\theta} n_r + t_{(1)\theta\theta} n_\theta /_{\mathcal{S}} = t_{(2)r\theta} n_r + t_{(2)\theta\theta} n_\theta /_{\mathcal{S}}. \tag{3.8}$$

By introducing (2.11) in (3.7) one gets

$$u_{(1)}(\beta) = u_{(2)}(\beta), \tag{3.9}$$

$$\varphi_{(1)}(\beta) = \frac{Bg_{(2)}}{Bg_{(1)}} \varphi_{(2)}(\beta), \tag{3.10}$$

$$\beta_1 = \frac{\frac{Bg_{(2)}}{Bg_{(1)}} v_{(2)}(\beta) - v_{(1)}(\beta)}{u'_{(1)}(\beta) - u'_{(2)}(\beta)}. \tag{3.11}$$

Since we have $\mathcal{S} \cap \mathcal{S}_{(2)j} = \{P_j\}$, $j=1,2$, there follows

$$\Psi^{\circ}_{(2)o}(\alpha) - \Psi^{\circ}_{(2)o}(\beta) = \frac{1}{2}\left(1 - \frac{R_4^2}{R_2^2}\right), \tag{3.12}$$

$$\beta_1 \Psi^{\circ\prime}_{(2)o}(\beta) + \frac{Bg_{(2)}}{Bg_{(1)}} \varphi_{(2)}(\beta) = 0. \tag{3.13}$$

The conditions that $\mathcal{S}_{(1)j}$ contains the points P_j, $j=1,2$ determines the values of the $C_{(1)j}$ constants, so that the equations of $\mathcal{S}_{(1)j}$ finally become

$$R_2^2 v_2 \left[\Psi^{\circ}_{(1)0}(\theta)-\Psi^{\circ}_{(1)0}(\beta)\right]+\frac{k_{(1)}}{\eta_{(1)}}\left[r^{3}_{(1)j}(\theta)\varphi(\theta)-\right.$$
$$\left.-r^{3}_{(1)j}(\beta)\varphi_{(1)}(\beta)\right]-\frac{k_{(1)}}{\eta_{(1)}}\beta_1\Psi^{\circ\prime}_{(1)0}(\beta)r^{3}_{(1)j}(\beta)- \tag{3.14}$$
$$-\frac{v_j}{2}\left[r^{3}_{(1)j}(\theta)\sin^2\theta-R^2_{3,4}\right]=0,\quad j=1,2.$$

By putting in (3.14) $\theta=0$, one gets

$$\Psi^{\circ}_{(1)0}(\beta)-\Psi^{\circ}_{(1)0}(0)=\frac{1}{2}\,\frac{R_4^2}{R_2^2}, \tag{3.15}$$

$$\varphi_{(1)}(\beta)+\beta_1\Psi^{\circ\prime}_{(1)0}(\beta)=0. \tag{3.16}$$

The last relation according to (3.9), (3.10) and (2.12) is identical to (3.13).

4. CONDITIONS ON THE DYE WALL AND CONTINUITY OF STRESS ON $\mathcal{S}$

On the die surface $\theta=\alpha$ we have $v_{(2)\theta}=0$ and the friction is described by the relation

$$t_{(2)r\theta}\Big|_{\theta=\alpha}=m\sqrt{\mathrm{II}\,t'_{(2)}}\Big|_{\theta=\alpha},$$

where m is the friction factor, satisfying the condition $0<m<1$. These boundary conditions imply [1 - 7]

$$\varphi_{(2)}(\alpha)=0 \tag{4.1}$$

and

$$u'_{(2)}(\alpha)=-2\sqrt{3}\,\gamma\,u_{(2)}(\alpha), \tag{4.2}$$

$$v'_{(2)}(\alpha)=\sqrt{3}\,\gamma\,v_{(2)}(\alpha), \tag{4.3}$$

where

$$\gamma=\frac{m}{\sqrt{1-m^2}}. \tag{4.4}$$

From (3.8) one also obtains

$$a_{(2)}=\frac{\eta_{(1)}}{\eta_{(2)}}a_{(1)},\quad c_{(2)}=\frac{\eta_{(1)}}{\eta_{(2)}}c_{(1)}, \tag{4.5}$$

$$u'_{(2)}(\beta)=\frac{\eta_{(1)}}{\eta_{(2)}}u'_{(1)}(\beta), \tag{4.6}$$

$$v'_{(1)}(\beta)-\frac{k_{(2)}}{k_{(1)}}v'_{(2)}(\beta)-12\,\frac{u_{(2)}(\beta)}{u'_{(2)}(\beta)}\left[\frac{Bg_{(2)}}{Bg_{(1)}}v_{(2)}(\beta)-\right. \tag{4.7}$$
$$\left.-v_{(1)}(\beta)\right]=$$

$$= \frac{\eta_{(2)}}{\eta_{(1)}} \frac{u'_{(2)}(\beta)}{2}\left[\frac{Bg_{(2)}}{Bg_{(1)}} \frac{1}{\sqrt{3u^2_{(2)}(\beta) + u'^2_{(2)}(\beta)/4}} - \frac{1}{\sqrt{3u^2_{(2)}(\beta) + \frac{\eta^2_{(2)}}{\eta^2_{(1)}} \frac{u'^2_{(2)}(\beta)}{4}}}\right]$$

$$k_{(1)} A_{(1)} = k_{(2)} A_{(2)} \tag{4.8}$$

and

$$\frac{k_{(2)}}{k_{(1)}} h_{(2)}(\beta) - h_{(1)}(\beta) + 4\left[v_{(1)}(\beta) - \frac{k_{(2)}}{k_{(1)}} v_{(2)}(\beta) -\right.$$

$$- 6\left(1 - \frac{\eta_{(2)}}{\eta_{(1)}}\right) \frac{\cot g\,\beta}{\sin\beta} \varphi_{(1)}(\beta) - u_{(2)}(\beta)\left[\frac{1}{\sqrt{3u^2_{(2)}(\beta) + \frac{\eta^2_{(2)}}{\eta^2_{(1)}} \frac{u'^2_{(2)}(\beta)}{4}}} - \right. \tag{4.9}$$

$$\left. - \frac{k_{(2)}}{k_{(1)}} \frac{1}{\sqrt{3u^2_{(2)}(\beta) + \frac{u'^2_{(2)}(\beta)}{4}}}\right] = 0.$$

5. THE DRAWING FORCE

Let Σ be a surface of equation $r = r(\theta)$, $\theta \in [0, \alpha]$ located within the deformation zone; we shall compute the stress resultant which acts on this surface. As in [1 - 7] we get

$$X = Y = 0,$$

$$Z = 2\pi \int_0^{\theta_s(r)} [r^2(\theta) \cos\theta \cdot t_{(1)rr} + r(\theta) r'(\theta) \sin\theta \cdot t_{(1)\theta\theta} -$$

$$- (r(\theta) r'(\theta) \cos\theta + r^2(\theta) \sin\theta) t_{(1)r\theta}] \sin\theta \, d\theta + \tag{5.1}$$

$$+ 2\pi \int_{\theta_{s(r)}}^{\alpha} [r^2(\theta) \cos\theta \, t_{(2)rr} + r(\theta) r'(\theta) \sin\theta \cdot t_{(2)\theta\theta} -$$

$$- (r(\theta) r'(\theta) \cos\theta + r^2(\theta) \sin\theta) t_{(2)r\theta}] \sin\theta \, d\theta,$$

where X, Y, Z are the components of the stress resultant in the frame Oxyz.

By substituting here (2.17), one obtains

$$Z = 2\pi\left[- \eta_{(1)} v_2 R_2^2 \frac{u'_{(1)}(\theta_s) \sin\theta_s \cos\theta_s}{r(\theta_s)} + \right.$$

$$+ \eta_{(2)} v_2 R_2^2 \frac{u'_{(2)}(\theta_s) \sin\theta_s \cos\theta_s}{r(\theta_s)} + 2\eta_{(2)} v_2 R_2^2 a_{(2)} \frac{\sin^2\alpha}{r(\alpha)} -$$

$$\left. - c_{(2)} \frac{\eta_{(2)} v_2}{2R_2} r^2(\alpha) \sin^2\alpha - \eta_{(2)} v_2 R_2^2 \frac{u'_{(2)}(\alpha) \sin\alpha \cos\alpha}{r(\alpha)}\right] +$$

$$+ 2\pi k_{(1)}\left\{- \frac{r^2(\theta_s) \sin^2\theta_s}{2}\left[h_{(1)}(\theta_s) + \frac{u_{(1)}(\theta_s)}{\sqrt{3u^2_{(1)}(\theta_s) + \frac{u'^2_{(2)}(\theta_s)}{4}}}\right] + \right. \tag{5.2}$$

$$+ \frac{r^2(\theta_s) \sin 2\theta_s}{\ldots} \frac{u'_{(1)}(\theta_s)}{\ldots} + 2 r^2(\theta_s) v_{(1)}(\theta_s) \sin^2\theta_s +$$

$$+ \frac{r^2(\theta_s)}{4} v'_{(1)}(\theta_s)\sin 2\theta_s - 3r^2(\theta_s)\varphi_{(1)}(\theta_s)\cos\theta_s \Big\} +$$

$$+ 2\pi k_{(2)} \Big\{ \frac{r^2(\theta_s)\sin^2\theta_s}{2}\Big[h_{(2)}(\theta_s) + \frac{u_{(2)}(\theta_s)}{\sqrt{3u^2_{(2)}(\theta_s) + \frac{u'^2_{(2)}(\theta_s)}{4}}}\Big] -$$

$$- \frac{r^2(\theta_s)\sin 2\theta_s}{8} \frac{u'_{(2)}(\theta_s)}{\sqrt{3u^2_{(2)}(\theta_s) + \frac{u'^2_{(2)}(\theta_s)}{4}}} - 2r^2(\theta_s)v_{(2)}(\theta_s)\sin^2\theta_s -$$

$$- \frac{r^2(\theta_s)}{4} v'_{(2)}(\theta_s)\sin 2\theta_s + 3r^2(\theta_s)\varphi_{(2)}(\theta_s)\cos\theta_s +$$

$$+ A_{(2)}\Big(\frac{r^2(\alpha)\sin^2\alpha}{2}\ln\frac{r(\alpha)}{R_2} - \frac{r^2(\alpha)\sin^2\alpha}{4}\Big) -$$

$$- \frac{r^2(\alpha)\sin^2\alpha}{2}\Big[h_{(2)}(\alpha) + \Big(u_{(2)}(\alpha) -$$

$$- \frac{u'_{(2)}(\alpha)}{2}\cot g\,\alpha\Big)\frac{1}{\sqrt{3u^2_{(2)}(\alpha) + \frac{u'^2_{(2)}(\alpha)}{4}}}\Big] +$$

$$+ 2r^2(\alpha)\sin^2\alpha\Big(v_{(2)}(\alpha) + \frac{\cot g\,\alpha}{4} v'_{(2)}(\alpha)\Big)\Big\}.$$

By denoting

$$Z^{I} = Z\Big/_{\mathcal{S}_{(1)1}\cup\mathcal{S}_{(2)1}}, \quad Z^{II} = -Z\Big/_{\mathcal{S}_{(1)2}\cup\mathcal{S}_{(2)2}} \tag{5.3}$$

and by using the conditions (3.9), (4.5), (4.6),(4.7) and (4.9) one gets

$$\begin{aligned} Z^{I} = 2\pi\Big[& 2\eta_{(2)}v_2R_2^2a_{(2)}\frac{\sin^3\alpha}{R_1} - c_{(2)}\frac{\eta_{(2)}v_2}{2R_2}R_1^2 - \\ & - \eta_{(2)}v_2R_2^2\frac{u'_{(2)}(\alpha)\sin^2\alpha\cos\alpha}{R_1}\Big] + \\ & + \pi k_{(2)}R_1^2\Big\{ A_{(2)}\Big(\ln\frac{R_1}{R_2\sin\alpha} - \frac{1}{2}\Big) - h_{(2)}(\alpha) - \\ & - \frac{1}{\sqrt{3u^2_{(2)}(\alpha) + \frac{u'^2_{(2)}(\alpha)}{4}}}\Big(u_{(2)}(\alpha) - \frac{u'_{(2)}(\alpha)}{2}\cot g\,\alpha\Big) + \\ & + 4v_2(\alpha) + v'_{(2)}(\alpha)\cot g\,\alpha\Big\}, \end{aligned} \tag{5.4}$$

$$Z^{II} = -2\pi\Big[2\eta_{(2)} v_2 R_2^2 a_{(2)} \frac{\sin^3\alpha}{R_2} - c_{(2)} \frac{\eta_{(2)} v_2 R_2}{2} -$$
$$- \eta_{(2)} v_2 R_2 u'_{(2)}(\alpha) \sin^2\alpha \cos\alpha\Big] -$$
$$- \pi k_{(2)} R_2^2 \Big\{A_{(2)}\Big(\ln\frac{1}{\sin\alpha} - \frac{1}{2}\Big) - h_{(2)}(\alpha) -$$
$$- \frac{1}{\sqrt{3u_{(2)}^2(\alpha) + \frac{u'^2_{(2)}(\alpha)}{4}}}\Big(u_{(2)}(\alpha) - \tag{5.5}$$
$$- \frac{u'_{(2)}(\alpha)}{2}\operatorname{cotg}\alpha\Big) + 4v_{(2)}(\alpha) + v'_{(2)}(\alpha)\operatorname{cotg}\alpha\Big\}.$$

If $\vec{T}$ and $\vec{N}$ are the stress resultants acting on the die surface $\theta=\alpha$, $r_{(2)2}(\alpha) \leq r \leq r_{(2)1}(\alpha)$, then

$$\vec{T} = \pi \sin 2\alpha\, \vec{k} \int_{r_{(2)2}(\alpha)}^{r_{(2)1}(\alpha)} t_{(2)r\theta}\Big|_{\theta=\alpha} r\,dr,$$

$$\vec{N} = -2\pi \sin^2\alpha\, \vec{k} \int_{r_{(2)2}(\alpha)}^{r_{(2)1}(\alpha)} t_{(2)\theta\theta}\Big|_{\theta=\alpha} r\,dr. \tag{5.6}$$

By means of (2.17) one obtains

$$\vec{T} = 2\pi\Big\{\eta_{(2)} v_2 R_2^2\Big(\frac{1}{R_1} - \frac{1}{R_2}\Big)\sin^2\alpha\cos\alpha \cdot u'_{(2)}(\alpha) -$$
$$- \frac{k_{(2)}}{2}\operatorname{cotg}\alpha\Big[v'_{(2)}(\alpha) + \tag{5.7}$$
$$+ \frac{u'_{(2)}(\alpha)}{\sqrt{3u_{(2)}^2(\alpha) + \frac{u'^2_{(2)}(\alpha)}{4}}}\Big](R_1^2 - R_2^2)\Big\}\vec{k},$$

$$\vec{N} = 2\pi\Big\{-2\eta_{(2)} v_2 R_2^2\Big(\frac{1}{R_1} - \frac{1}{R_2}\Big)a_{(2)}\sin^2\alpha +$$
$$+ \frac{\eta_{(2)} v_2}{2R_2} c_{(2)}(R_1^2 - R_2^2) - \tag{5.8}$$
$$- \frac{k_{(2)}}{2} R_1^2\Big[A_{(2)}\Big(\ln\frac{R_1}{R_2\sin\alpha} - \frac{1}{2}\Big) -$$
$$- \frac{u_{(2)}(\alpha)}{\sqrt{3u_{(2)}^2(\alpha) + \frac{u'^2_{(2)}(\alpha)}{4}}} + 4v_{(2)}(\alpha) - h_{(2)}(\alpha)\Big] +$$

$$+\frac{k_{(2)}}{2}R_2^2\left[A_{(2)}\left(\ln\frac{1}{\sin\alpha}-\frac{1}{2}\right)-\frac{u_{(2)}(\alpha)}{\sqrt{3u_{(2)}^2(\alpha)+\frac{u'^2_{(2)}(\alpha)}{4}}}+\right.$$

$$\left.+4v_{(2)}(\alpha)-h_{(2)}(\alpha)\right]\Big\}\vec{k}.$$

One can easily verify that

$$\overrightarrow{Z^{I}}+\overrightarrow{Z^{II}}+\vec{T}+\vec{N}=\vec{0},$$

that is, the resultant of the forces acting upon the deformation zone boundary $\mathcal{D}$ is zero.

By denoting with

$$\sigma_{z1}=\frac{Z^{I}}{\pi R_1^2},\quad \sigma_{z2}=\frac{Z^{II}}{\pi R_2^2}, \tag{5.9}$$

from (5.4) and (5.5) there results

$$\sigma_{z1}+\sigma_{z2}=2\eta_{(2)}v_2R_2^2\left(\frac{1}{R_1^3}-\frac{1}{R_2^3}\right)\sin^2\alpha\left[2a_{(2)}\sin\alpha-\right.$$
$$\left.-u'_{(2)}(\alpha)\cos\alpha\right]+k_{(2)}A_{(2)}\ln\frac{R_1}{R_2}. \tag{5.10}$$

If we divide the stresses by $\sigma_{(2)Y}=k_{(2)}\sqrt{3}$, one gets the relative drawing stress

$$\frac{|\sigma_{z2}|}{\sigma_{(2)Y}}=\frac{\sigma_{z1}}{\sigma_{(2)Y}}+\frac{2}{\sqrt{3}\,Bg_{(2)}}\sin^2\alpha\left[2a_{(2)}\sin\alpha-\right.$$
$$\left.-u'_{(2)}(\alpha)\cos\alpha\right]\left(1-\frac{R_2^3}{R_1^3}\right)-\frac{A_{(2)}}{\sqrt{3}}\ln\frac{R_1}{R_2}. \tag{5.11}$$

6. NUMERICAL EXAMPLES

In order to determine the constants $a_{(i)}$, $b_{(i)}$, $c_{(i)}$, $d_{(2)}$ and the angle β, we use the equations (3.9), (3.12), (3.15), (4.2 (4.5), (4.6) and (5.4) where we assume the drawing back-force as known. The relative drawing stress given by (5.11) being the most important global characteristic of the process, the parameters involved in its are determined as follows: assuming that $\overset{\circ}{\psi}_{(1)o}(0)=0$, the constants $a_{(2)}$, $b_{(2)}$, $d_{(2)}$ and the angle β result from the following system

$$6a_{(2)}(1-\cos\beta)-b_{(2)}p'(\beta)\sin\beta-d_{(2)}q'(\beta)\sin\beta=$$
$$=3\,\frac{\eta_{(1)}}{\eta_{(2)}}\,\frac{R_3^2}{R_1^2},$$

$$6a_{(2)}(\cos\alpha-\cos\beta)+b_{(2)}\left[p'(\alpha)\sin\alpha-p'(\beta)\sin\beta\right]+$$

$$+ d_{(2)}\left[q'(\alpha)\sin\alpha - q'(\beta)\sin\beta\right] = -3\left(1-\frac{R_3^2}{R_1^2}\right),$$

$$a_{(2)}\left(1-\frac{\eta_{(1)}}{\eta_{(2)}}\right) + b_{(2)}\left(1-\frac{\eta_{(1)}}{\eta_{(2)}}\right)p(\beta) +$$

$$+ d_{(2)}\left[\frac{q'(\beta)p(\beta)}{p'(\beta)} - \frac{\eta_{(1)}}{\eta_{(2)}}q(\beta)\right] = 0, \tag{6.1}$$

$$2\sqrt{3}\,\gamma\, a_{(2)} + b_{(2)}\left[p'(\alpha) + 2\sqrt{3}\,\gamma\, p(\alpha)\right] +$$

$$+ d_{(2)}\left[q'(\alpha) + 2\sqrt{3}\,\gamma\, q(\alpha)\right] = 0.$$

To determine the angle β we use the transcendent equation which results from the compatibility of the system (6.1). We also mention the following relation

$$u(\theta) = \frac{\eta_{(2)}}{\eta_{(1)}}u_{(2)}(\theta) + \frac{9}{16}u_{(2)}(\beta)\sin\beta\cdot\left(1-\right.$$

$$\left. - \frac{\eta_{(2)}}{\eta_{(1)}}\right)\left(q'(\beta)p(\theta) - q(\theta)p'(\beta)\right). \tag{6.2}$$

In order to determine the constants $A_{(i)}$, $B_{(i)}$, $D_{(2)}$, $h_{(1)}(0)$, $h_{(2)}(\beta)$, $\varphi_{(2)}(\beta)$ and β_1 we use the equations (3.10), (3.11), (3.13), (4.1), (4.3) , (4.7), (4.8); (4.9) and (5.5). By different combinations of these relations one gets the following system to determine $A_{(2)}$, $B_{(2)}$ and $D_{(2)}$ which are present in (5.11)

$$A_{(2)}\frac{\gamma\sqrt{3}}{6} + B_{(2)}\left[\gamma\sqrt{3}\,p(\alpha) - p'(\alpha)\right] + D_{(2)}\left[\gamma\sqrt{3}\,q(\alpha) - q'(\alpha)\right] =$$

$$= -K_{(2)1}(\alpha)\left[\gamma\sqrt{3}\,p(\alpha) - p'(\alpha)\right] - K_{(2)2}(\alpha)\left[\gamma\sqrt{3}\,q(\alpha) - q'(\alpha)\right],$$

$$A_{(2)}\left\{\frac{p'(\beta) + 12p(\beta)\dfrac{u_{(2)}(\beta)}{u'_{(2)}(\beta)}}{p'(\beta)\sin\beta}\left[1-\cos\beta - \frac{\eta_{(1)}}{\eta_{(2)}}(\cos\alpha - \cos\beta)\right] + \right.$$

$$\left. + 2\frac{u_{(2)}(\beta)}{u'_{(2)}(\beta)}\left(1-\frac{\eta_{(1)}}{\eta_{(2)}}\right)\right\} - B_{(2)}\left\{\left(1-\frac{\eta_{(1)}}{\eta_{(2)}}\right)p'(\beta) + \right.$$

$$\left. + \frac{\eta_{(1)}}{\eta_{(2)}}\frac{p'(\alpha)\sin\alpha}{p'(\beta)\sin\beta}\left[p'(\beta) + 12\frac{u_{(2)}(\beta)p(\beta)}{u'_{(2)}(\beta)}\right]\right\} -$$

$$- D_{(2)}\left\{\left(1-\frac{\eta_{(1)}}{\eta_{(2)}}\right)q'(\beta) + \frac{\eta_{(1)}}{\eta_{(2)}}\frac{q'(\alpha)\sin\alpha}{p'(\beta)\sin\beta}\left[p'(\beta) + \right.\right.$$

$$\left.\left. + 12\frac{u_{(2)}(\beta)p(\beta)}{u'_{(2)}(\beta)}\right] - \frac{64}{3}\frac{\eta_{(1)}}{\eta_{(2)}}\frac{u_{(2)}(\beta)}{u'_{(2)}(\beta)p'(\beta)\sin\beta}\right\} = \tag{6.3}$$

$$= -6\frac{\eta_{(1)}}{\eta_{(2)}}\frac{p'(\beta) + 12p(\beta)\dfrac{u_{(2)}(\beta)}{u'_{(2)}(\beta)}}{p'(\beta)\sin\beta}K_{(2)}(\alpha) -$$

$$- \frac{k_{(1)}}{k_{(2)}} \frac{p'(\beta) + 12\, p(\beta) \dfrac{u_{(2)}(\beta)}{u'_{(2)}(\beta)}}{p'(\beta) \sin\beta} \int_0^\beta f_{(1)}(\theta) \sin\theta \, d\theta +$$

$$+ \frac{64}{3} \frac{k_{(1)}}{k_{(2)}} \frac{u_{(2)}(\beta)}{u'_{(2)}(\beta)\, p'(\beta) \sin\beta} K_{(1)2}(\beta) +$$

$$+ \frac{\eta_{(2)}}{\eta_{(1)}} \frac{u'_{(2)}(\beta)}{2} \left[\frac{\eta_{(1)}}{\eta_{(2)}} \frac{1}{\sqrt{3u^2_{(2)}(\beta) + \dfrac{u'^2_{(2)}(\beta)}{4}}} - \right.$$

$$\left. - \frac{k_{(1)}}{k_{(2)}} \frac{1}{\sqrt{3u^2_{(2)}(\beta) + \dfrac{\eta^2_{(2)}}{\eta^2_{(1)}} \dfrac{u'^2_{(2)}(\beta)}{4}}} \right],$$

$$A_{(2)} \left[\cos\alpha - \cos\beta + \frac{6\, u_{(2)}(\beta)\, p(\beta)}{u'_{(2)}(\beta)\, p'(\beta)} \left(\cos\beta - \right. \right.$$

$$\left. \left. - \frac{1 - \dfrac{\eta_{(1)}}{\eta_{(2)}} \cos\alpha}{1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}} \right) - \frac{u_{(2)}(\beta) \sin\beta}{u'_{(2)}(\beta)} \right] + B_{(2)} \left[p'(\alpha) \sin\alpha - \right.$$

$$\left. - p'(\beta) \sin\beta + 6 \frac{\eta_{(1)}}{\eta_{(2)}} \frac{u_{(2)}(\beta)\, p(\beta)\, p'(\alpha) \sin\alpha}{\left(1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}\right) u'_{(2)}(\beta)\, p'(\beta)} \right] +$$

$$+ D_{(2)} \left[q'(\alpha) \sin\alpha - q'(\beta) \sin\beta + \right.$$

$$+ 6 \frac{\eta_{(1)}}{\eta_{(2)}} \frac{u_{(2)}(\beta)\, p(\beta)\, q'(\alpha) \sin\alpha}{\left(1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}\right) u'_{(2)}(\beta)\, p'(\beta)} -$$

$$\left. - \frac{32}{3} \frac{\eta_{(1)}}{\eta_{(2)}} \frac{u_{(2)}(\beta)}{\left(1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}\right) u'_{(2)}(\beta)\, p'(\beta)} \right] =$$

$$= 6 \left[1 + 6 \frac{\eta_{(1)}}{\eta_{(2)}} \frac{u_{(2)}(\beta)\, p(\beta)}{\left(1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}\right) u'_{(2)}(\beta)\, p'(\beta)} \right] K_{(2)}(\alpha) +$$

$$+ 6 \frac{k_{(1)}}{k_{(2)}} \frac{u_{(2)}(\beta)}{\left(1 - \dfrac{\eta_{(1)}}{\eta_{(2)}}\right) u'_{(2)}(\beta)\, p'(\beta)} \left[- \frac{16}{9} K_{(1)2}(\beta) + \right.$$

$$\left. + p(\beta) \int_0^\beta f_{(1)}(\theta) \sin\theta \, d\theta \right],$$

where

$$K_{(2)}(\alpha) = -\frac{1}{6}\left[p'(\alpha)K_{(2)1}(\alpha) + q'(\alpha)K_{(2)2}(\alpha)\right]\sin\alpha + \tag{6.4}$$
$$+\frac{1}{6}\int_{\beta}^{\alpha} f_{(2)}(\theta)\sin\theta\, d\theta .$$

The material constants have been taken out of [8] for copper and aluminium, that is

$$k = \frac{110}{\sqrt{3}}\ \mathrm{MPa}, \quad \eta = 0{,}11\ \mathrm{MPa\cdot s},$$

and

$$k = \frac{70}{\sqrt{3}}\ \mathrm{MPa}, \quad \eta = 0{,}09\ \mathrm{MPa\cdot s}.$$

respectively.

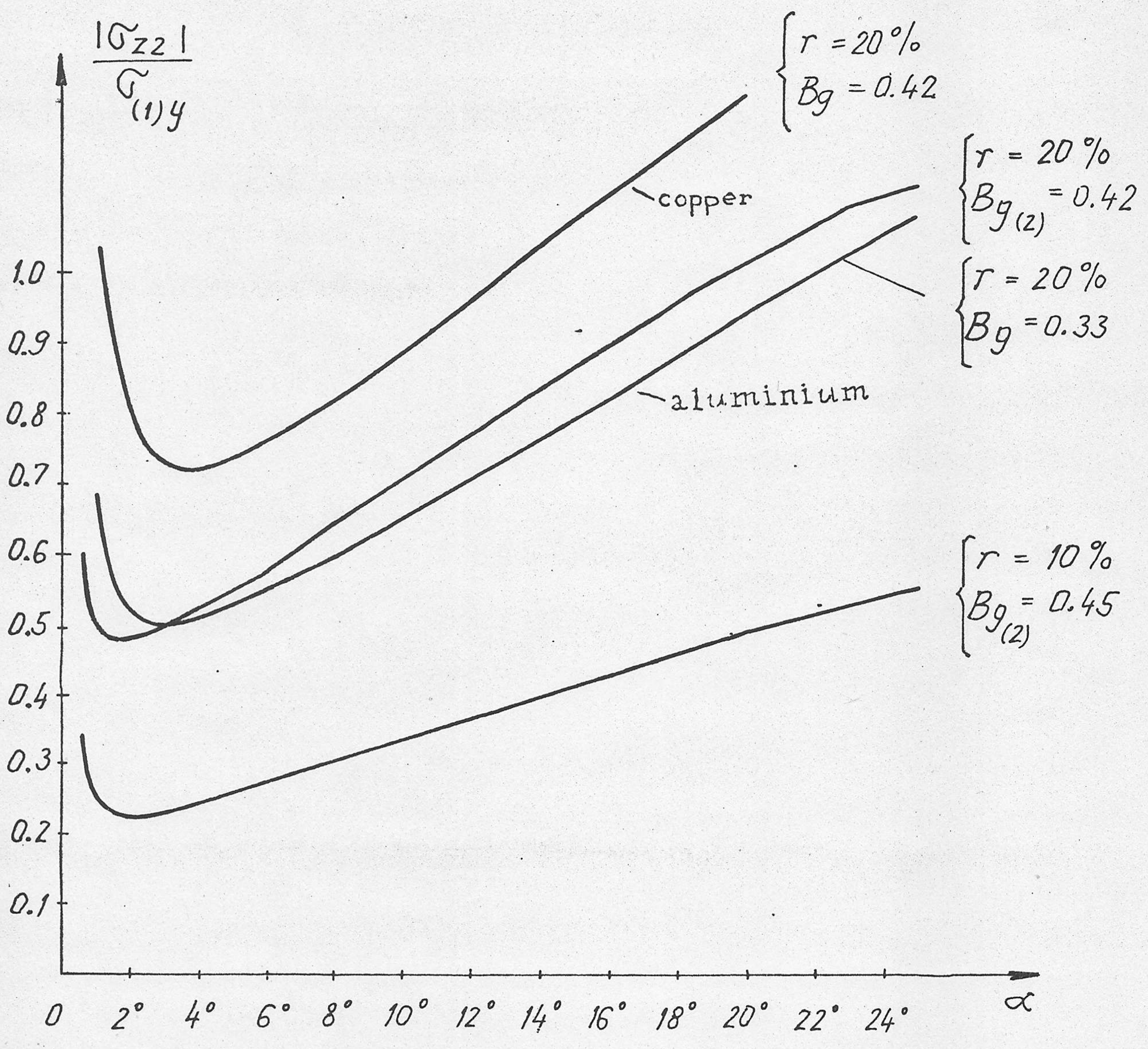

Fig. 2. The relative drawing stress, depending on the angle α , in case 1 .

The drawing velocity is $v_{(2)} = 1$ m/s . The geometrical data for thin wires have been considered as follows

$R_1 = 0,821$ mm., $R_3 = 0,677$ mm

and for reductions $r\% = 100(1 - R_2^2/R_1^2)$ of 10% and 20%, which corresponds to the following values of R_2

$R_2 = 0.7788$ mm and $R_2 = 0.7343$ mm

respectively.

Two cases have been considered:

Case 1 - wires with copper external cover and aluminium core for which $r\% = 10\%$, $Bg_{(2)} = 0.45$ and $r\% = 20\%$, $Bg_{(2)} = 0.42$.

Case 2 - wires with aluminium external cover and copper core for which $r\% = 10\%$, $Bg_{(2)} = 0.35$ and $r\% = 20\%$, $Bg_{(2)} = 0.33$.

For both cases, when $r\% = 20\%$, the drawing stress values have been compared with the corresponding ones for monometallic wires of copper and aluminium , respectively.

The numerical data obtained are illustrated in figures 2 and 3.

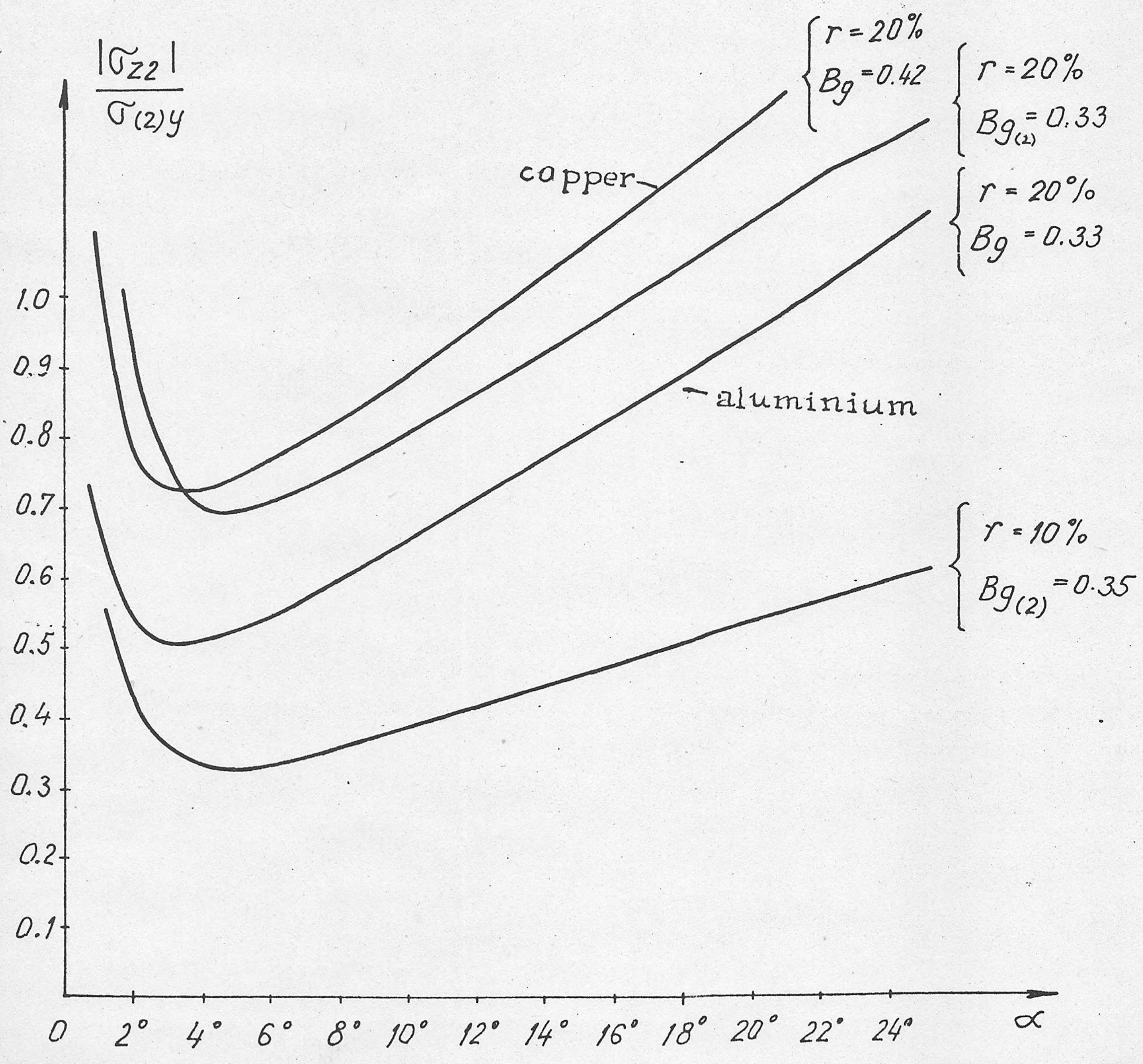

Fig. 3. The relative drawing stress, depending on the angle α , in case 2.

As mentioned above, the deformation zone geometry depends on the form of the separation surface $\mathcal{S}$ of the two components, whose equation is given by (3.5) and depends on the parameters β and β_1

In both cases the system (6.1) determines the parameter β as a function of the angle α. The numerical results are shown in fig. 4.

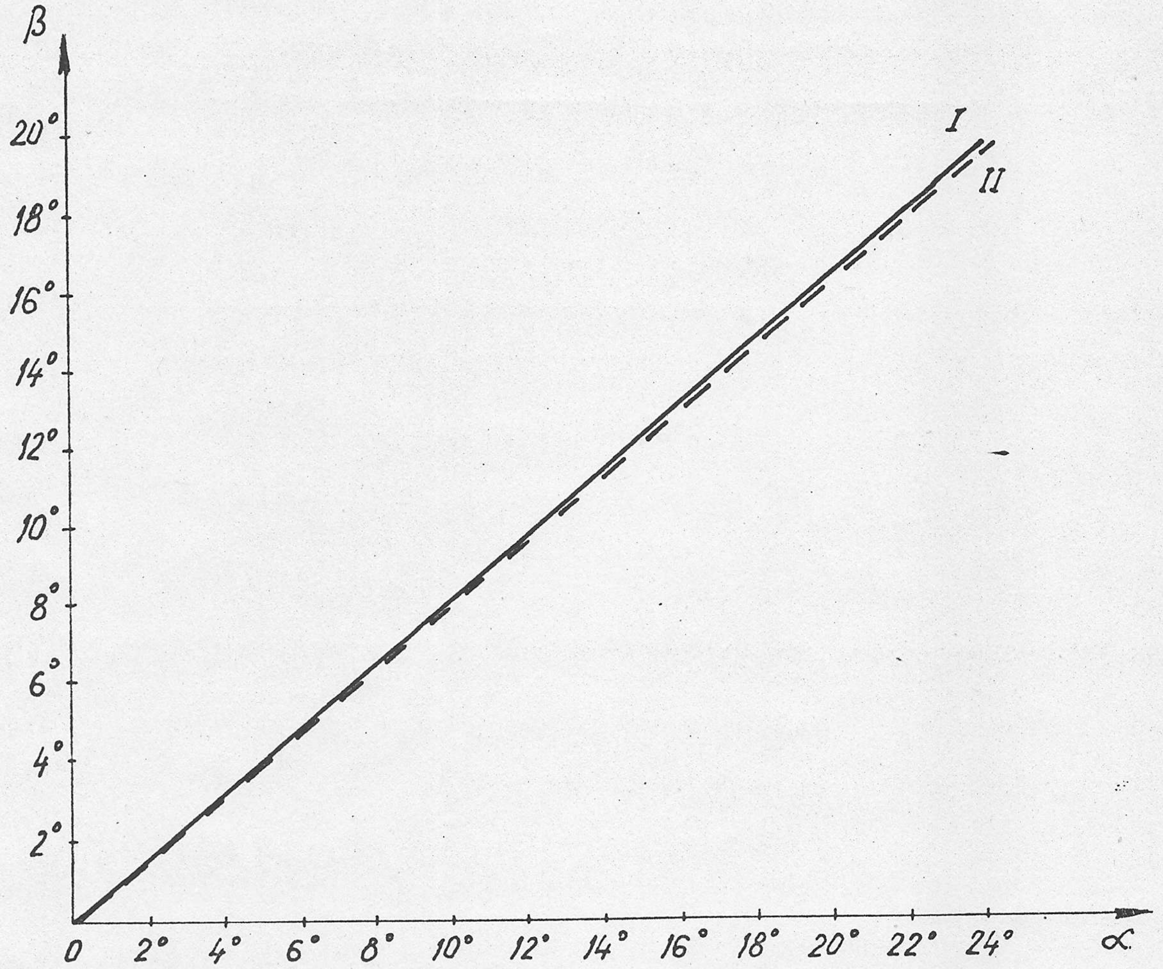

Fig. 4. The dependence of the parameter β on the angle α.

The parameter β_1 is determined from (3.11) as a function of the angle α, and it is graphically represented in fig. 5.

7. CONCLUSIONS

The drawing process of a wire with two Bingham type components through a conical converging die has been theoretically described and numerically exemplified. The influence of the drawing velocity on the drawing stress is given through the Bingam's number.

The existence of a optimal die angle for which the relative drawing stress reaches a minimum has also been established for bimetal drawing.

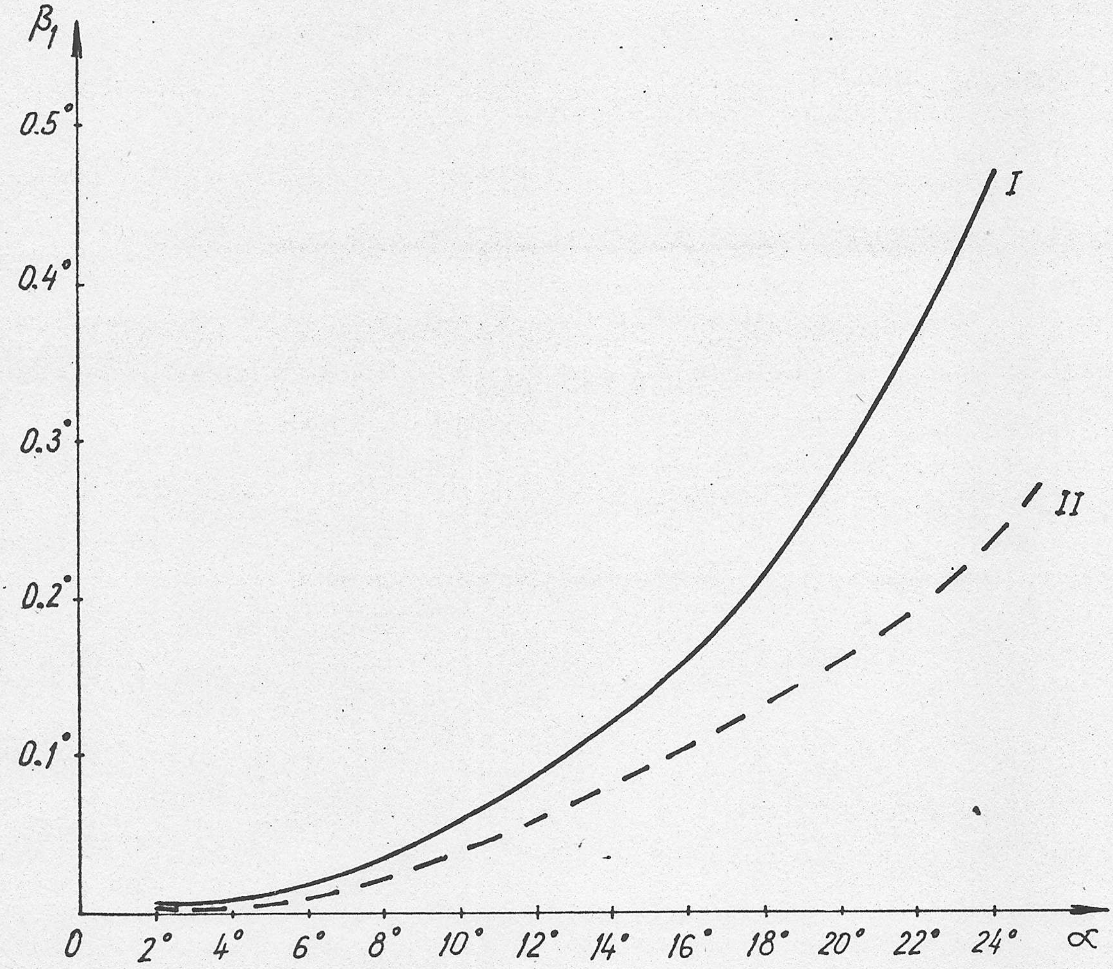

Fig. 5. The dependence of the parameter β_1 on the angle α .

This relative stress is located, as it was expected, between the two corresponding stresses of the monometallic wires.

The boundary between the two components of the bimetallic wire within the deformation zone has also been specified in the examples given above.

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