ELASTOVISCOPLASTIC MODELS WITH RELAXED CONFIGURATIONS AND INTERNAL STATE VARIABLES

by

SANDA CLEJA-TIGOIU*) and EUGEN SOOS**)

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*) Faculty of Mathematics, University of Bucharest, Str.Academiei 14, Bucharest 70109, Romania.

**) National Institute for Scientific and Technical Creation, Department of Mathematics, Bd.Păcii 220,79622,Bucharest, Romania.

A B S T R A C T

In the paper we present the microstructural basis, the initial macroscopical formulations, and a possible axiomatic reconstruction of the elastoviscoplastic model for metals, based on the use of the local current relaxed configurations.

The structural analysis and the experimental data show that the utilization of these configurations offers avantages for the formulation of the material laws, when the deformations are moderately large (30%).

We think our review paper constitute a concise, historical and critical exposition of the main stages, contributions and results, which led, during 1966-1972, to the formulation of the fundamental ideas lying at the basis of the model.

We hope that the paper shows clearly the role played by LEE, LIU, TEODOSIU, SIDOROFF, MANDEL and KRATOCHVIL in the first formulation of the theory between 1966-1972, and the contribution of DAFALIAS and LORET to the development of the model between 1983-1985.

The paper is divided into 5 chapters.

In the first one we present concisely the microstructural basis of the model.

In the second chapter, based on the papers due to LEE, LIU, TEODOSIU, SIDOROFF, MANDEL, KRATOCHVIL, HALPHEN and NOLL, we deal with the axiomatic presentation of the model together with the principal properties following from

the hypotheses and definitions adopted.

Chapter 3 is devoted to the analysis played by the plastic rotation and by the internal state variables in modelling the anisotropic hardening of the structural isotropic materials.

In the forst chapter, based on the results due to PIPKIN, RIVLIN, OWEN, SILHAVY, LUCHESI and PODIO-GUIDUIGLI, we present the model of the materials with elastic range, and we analyse the connection existing between the two models.

The last chapter presents some results obtained between 1985-1988 concerning models based on the local current relaxed configurations and which were published in International Journal of Plasticity. As we think, our discussion reveals the unsatisfactory and confusing situation existing in the theory today.

Content

Introduction

In this review paper we present briefly the microstructural basis, the initial phenomenological formulations, and, a possible axiomatic reconstruction of the elastoviscoplastic model for metals,based on the local current relaxed configurations concept. Both, the structural analysis based on the dislocations theory, and the convincing experimental data, show that the utilization of these configurations offers significant advantages for the formulation of the constitutive and evolution laws, when the deformations are large, but moderate.

In elaborating this paper we have tried to comply with the main A.M.R. recommendations: "Inclusion of original research should not per se invalidate an articole for A.M.R., but its <<review>> character should be dominant", and "a particular point of view or slant is okay if valid, openly stated, and interesting and useful".

In the framework of these recommendations, we think that our exposition constitutes a concise, historical and critical presentation of the main stages, contributions and results, which led, during 1966-1972, to the materialization of the fundamental ideas lying at the basis of the model.

We hope that the paper shows clearly and in a convincing way the role played by Lee, Liu, Teodosiu, Sidoroff, Mandel, Kratochvil in the first formulations of the theory between 1966-1972, and the important contribution of

Dafalias and Loret to the development of the model between 1983-1985.

Taking into account the first recommendation, we consider that the inclusion of our research regarding the axiomatic reconstruction of the model in this review constitutes a useful development of the theory dealt in this paper. The critical analysis carried out in this review, using results obtained from the emphasized axiomatic system and the properties inferred on the basis of the adopted hypotheses, has had several aims: a) to show explicitly the suppositions tacitly assumed by the founders of the model; b) to underline the constitutive nature of the existence of the local relaxed configurations; c) to find an adequate mathematical language in order to formulate the hypotheses based on the essential structural facts; d) to eliminate the ambiguities, vagueness, errors, suppositions and unacceptable conclusions which are present in the considerations of both those who founded the model and those who contributed to its development; e) to emphasize the physical basis and the experimental facts which motivate and justify the assumed hypotheses, and to outline in the process the limits of the applicability of the model; f) to point out the decisive role played by the set of local, current, relaxed, isoclinic configurations (l.c.r.i.c) in constructing the theory, in defining correctly the elastic and plastic deformations, in formulating the material laws,in underlying the fact that both the elastic and

and plastic deformations are not purely kinematic concepts, and their correct presentatin requires an adequate constitutive framework. The omission of this facts lies at the basis of the inaccurate formulations, erroneus interpretations and unacceptable suppositions present in many papers devoted to the model analysed here, such as: 1) disputes on the uniqueness or non-uniqueness of the current relaxed configurations; 2) the "proof" of the isotropy of all material functions (see for instance Sidoroff (1971), Casey, Naghdi (1980), (1981), Dafalias(1983), (1985)); 3) removing the "non-uniqueness" on the basis of generally unacceptable criteria, with a limited domain of applicability (see for instance Lee (1969), Lee, McMecking (1980), Lubarda, Lee (1981)).

We hope that our axiomatic reconstruction succeedes in showing convincingly the unacceptability of the facts underlined at 1)-3).

In this sense, we belive that our paper answers the second recommendation, i.e. to be useful and interesting to those who want to understand the essence of the model.

The paper is divided into 5 chapters.

In the first one, we present concisely the microstructural basis of the model, the micro-and macroarguments which lie at the basis of the introduction of elastic and plastic deformations together with the specification of the constitutive framework.

In the second chapter, based on the papers due to Lee, Liu (1967), (1968), Lee (1969), (1970), Teodosiu(1970), (1975), Sidoroff (1970),(1971), (1973), (1974), Mandel (1971), (1972),(1973),(1974),(1982), Kratochvil (1971),(1972),(1974), Halphen (1975), Teodosiu, Sidoroff (1976), we deal with the axiomatic reconstruction of the model together with the principal properties following from the hypotheses and definitions adopted here.

Chapter 3 is devoted to the analysis of the role played by the plastic rotation and the internal state variables (i.v.s.) in modelling the anisotropic hardering of the structural isotropic materials. The analysis outlines the limits of the various models.

In the forth chapter, based on the results due to Pipkin, Rivlin (1971), Owen (1968),(1970),(1974), Silhavy (1977), Luccesi, Podio-Guidugli (1986), we briefly present the model of the materials with elastic range, and we analyse the connection between their model and our model.

The last chapter presents some results obtained between 1985-1988 concerning models based on the local current relaxed configurations and which were published in Int.J.of Plasticity. The discussion reveals the unsatisfactory and confusing situation existing in the theory.

In the paper we do not deal with the problems related to the existence of viscoplastic potential (see for instance Rice (1971), Mandel (1971d), Nguyen, Halphen (1973), (1975), Teodosiu, Sidoroff (1976)).

The refference list is not exhaustive, but it contains, we hope, the main contributions leading-during the last twenty five years - to the foundation of the analysed model.

Chap.1. The interference of the microstructure with the macrostructure

1.1. Physical basis; dislocations

In all what follows we denote by F, θ, α (or α_j), T and μ^e the deformation gradient, the absolute temperature, the set of the internal state variables, the Cauchy's stress and the shear modulus, respectively. The meaning of other symbols will be given simultaneously with their occurence.

The main problem of the macroscopic theories of plasticity is to establish the constitutive and evolution equations. Two types of theories have been developed (1) the history type and (2) the internal state type (see Kröner, Teodosiu (1972)):

1) In the history type theories the material is specified by a response functional, of a priori unknown form, usually taken as a local time functional of the strain. An essential difficulty arises (in all these theories): in order to obtain the unknown functional from experiments, one must apply in these experiments, all possible load histories. Therefore the specification of such functionals is practically an almost impossible task.

2) "It is therefore, preferable to replace the past history (say, for instance) of F and θ by the present values of some internal variables $\alpha(t)$, generally a set of scalars and/or tensors, which should account, in a condensed

and simplified way, either for the past inelastic history of F and θ, or for the current structural arrangement it has produced at the microscale" (Teodosiu, Sidoroff (1986)).

The complex behaviour of metals derives from the complexity of their microstructural rearrangement and consequently any phenomenological theory must contain microstructural informations, relevant for certain classes of materials and deformation processes and able to acheive a simplified macroscopic description of the microstructural physical mechanisms.

The interference of the microstructure with the macrostructure can be acheived, for instance in the framework of the models with internal state variables, elaborated for metals. The internal state variables are mathematically described by scalar, vectorial or tensorial fields, and their present values replace, in a simplified manner, the dependence on the history of the deformation (or of the stress) and of the temperature.

The problem is to identifie the internal state variables with a sufficiently small number of parameters, which would be relevant for the microstructural rearrangement (see for instance Kröner (1958), (1963), (1970), Kröner, Teodosiu (1972), Teodosiu (1982), Sidoroff, Teodosiu (1986).

"The physical research on plastic and viscoplastic, i.e. crystalline materials, has revealed the existence of an internal mechanical state which is the lattice defect state. The quantities used for the description of this state are called internal quantities. From the phenomenological standpoint they are hidden quantities because devices on a microscopic scale are necessary to make these quantities visible. We call internal state variables those quantities which vary during the experiments, in contrast to internal state parameters, which also specify the internal state, but remain constant during the deformation" (Kröner, Teodosiu (1972)).

"A theory which claims to be a physical theory must not leave open the physical meaning of the internal variables. It is above all the physical identification of these variables which fills the frame of the theory with physical live" (Kröner, Teodosiu (1972)). When dealing with plasticity the most important variable part of the internal state is the dislocation state.

The physical research shows that the plasticity and the viscoplasticity are typical properties of the crystalline materials and evidence the fact that the defects (dislocations, point defects, grains, etc.) are the principal factors of certain elementary processes which can be observed at macroscopic level via the permanent deformations produced. From among the defects of the crystalline structure, the dislocations are those which by their motion and generation produce the plastic permanent deformations and involve changes of the internal mechanical structure during the deformation process.

If during deformation the distribution of the dislocations alone varies, then as internal state variables involved in the theory we choose those which specify the dislocation arrangement.

In the dislocation theory, a domain belonging to the physics of solids, the connection between the plastic properties of crystals and their atomic structure is investigated.

The starting point of this theory is linked to the papers of Taylor, Orowan and Polányi in the early thirties which attempted to elucidate the atomic mechanism of the slip in crystals.

The fundamental aspects of the slip, as given by Cottrell (1964) are:

a) One side of the crystal, as a whole, slips on a slip surface with respect to the other part, along a determined direction, called the slip direction. Often this surface is a plane and, then, it is called the slip plane.

b) The distance between the slip planes and the amount of the slip produced in various planes are usually in the range $10^{-6} - 10^{-4}$ cm and $10^{-7}-10^{-5}$ cm, respectively.

c) The slip direction always coincides (practically) with the direction of the lattice vector situated on the plane of maximum packing.

d) Frequently, but not always, the slip plane is that in which the packing is maximum, but one can observe smaller slips in other crystallografic planes, as well.

e) The slip in a system (slip plane and slip direction) begins when the tangential stress in the slip plane reaches a critical value, called the reduced tangential stress (the Schmid law). The influence of the stress component normal to the slip plane on the begining of the slip can be neglected within the usual limits of the standard experiments (stresses up to $10^{-2}\mu^{e}$).

f) The critical value of the reduced tangential stress is varying (for a given specimen) within large limits, depending on the temperature and on the rate of deformation. For instance, for pure metals (Cu, Al, Zn) it ranges between $10^{-5}\mu^{e} - 10^{-4}\mu^{e}$.

g) Through out the slip process on the slip plane the material retains its crystalline structure. This can be established by the fact that the slip takes always place mainly along the crystalline direction on the slip plane which corresponds to a maximum packing even if the tangential stress is not acting exactly in the corresponding direction. By contrast, if the metal is melted, which means that the crystalline structure is destroyed, then the slip is produced along the direction in which the tangential stress is maximum.

In a finite deformation theory of elastoplasticity of single crystals we must also take into account (see Teodosiu, Sidoroff (1976)) that:

h) Despite the high dislocation density, the region in which the crystalline structure is deteriorated, represents an infinitesimal percentage of the total volume.

This allows us to define a mean crystallografic orientation for each macroscopic volume element.

i) The dislocations passing through a volume element produce an irreversible permanent change of its shape (the viscoplastic deformation)and do not change significantly the mean orientation of the crystalline structure. In the case of the non-uniform viscoplastic deformations, the deformations of the volume elements are generally incompatible with each other and this brings about the elastic deformations and the residual stresses.

j) The self-stresses produced by the dislocations remaining inside a volume element hinder the motion of dislocations. This leads to the hardening of the material, phenomenon which is observed at a macroscopic level.

k) Although the elementary slip produced by each dislocation is discontinuous, when passing from one particle to the other, the plastic deformation, as well the elastic deformation, may be considered continuous at macroscopic level, due to the high dislocation density.

In the theories of plasticity the most important variable part of the internal mechanical state is the state of dislocations. Consequently, we must choose those variables which are able to describe this state at macroscopic level.

1.2. Elastic and plastic deformations

Further on, we present some of the considerations given by Teodosiu (1970) concerning the concepts of plastic and elastic deformations, and which were introduced on the basis of the physical and experimental argumetns a)-k).

We consider a single crystal $\mathcal{B}$ which at the moment t=0 is free of external loading and which is at the uniform absolute temperature Θ_o in its global reference configuration k_o. We assume that $\mathcal{B}$ contains defects, as dislocations, which may produce residual stresses (corresponding to (i)). Consequently, the body has not a stress-free global configuration.

Let X be an arbitrary particle of $\mathcal{B}$, and $\mathcal{N}_X$ a material neighbourhood of X. This neighbourhood is chosen small in comparison with dimension of $\mathcal{B}$ in k_o, but large with respect to the mean distance between defects (such type of neighbourhood exists according to (b)).

We <u>assume</u> that, at least in principle, we may cut out from the body this neighbourhood $\mathcal{N}_X$ and allow it to relax, mentaining constant both the temperature and the (relativ) position of dislocations which are contained in $\mathcal{N}_X$. We mention that in practice the dislocations configuration can be kept nearly constant by irradiating the crystals with fast neutrons before unloading (see for instance Teodosiu (1975)).

The configuration K_o obtained by such a procedure will be called the local relaxed configuration (l.r.c.) of $\mathcal{N}_X$ (see Fig.1).

Fig.1

We underline that the existence of the configuration K_o, in which the macroscopic stress is zero, is a <u>cons-</u>

tutive assumption.

Now we suppose that the single crystal is deformed under an external loading and, possible, under a non-uniform temperature field. Let k be a current configuration of $\mathcal{B}$ at time t. We denote by F the deformation gradient from the initial global configuration k_o to the current global configuration k.

We assume that, at least in principle, at any time t>0 we can repeate the local relaxation procedure used at the initial moment. We denote by K_t[1] the local relaxed configuration of $\mathcal{N}_X$ obtained at the moment t, by the following procedure: we cut out $\mathcal{N}_X$ from $\mathcal{B}$ by bringing the temperature back to its initial value, and by reducing instantaneously the macroscopic stress to zero, but keeping at the same time the (relativ) positions and values of all existing defects in $\mathcal{N}_X$ constant (Kröner, Teodosiu (1972)).

We also assume that the local relaxed configurations are defined except for a rigid rotation. (This hypothesis has an obviously intuitive character).

Since the reversible elastic deformation represents the deformation of the crystalline lattice (which remains unchanged by the dislocation motion, according to (h)) the indetermination in choosing the local current configuration has to be eliminated. We accomplish this task by assuming that in all local current relaxed configurations

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[1] Whenever confusions are not likely to occur we shall use K instead of K_t.

the crystalline directions are parallel to each other.

Once K_o is fixed, this criterion determines uniquelly the set of local current relaxed configurations,- apart from the ortogonal maps contained in the material symmetry group of the analised particle.

The configurations K_t constructed by this procedure will be called local, current, relaxed, isoclinic configurations (l.c.r.i.c.).

As pointed out by Kratochvil (1971), the use of the local, current, relaxed, isoclinic configuration ensures the invariance of the plastic deformation at a change of frame. The introduction of this crystallografic triad in order to define correctly, on a physically motivated basis, the elastic and plastic deformations is due to Teodosiu (1970), Mandel (1971), (1972a,b), (1973) and Kratochvil (1971).

According to the choice made the local deformation from K to k describes the deformation of the crystalline lattice (see (g), (h), (i)) and further "the glide directions and planes in the configuration K will be parallel to those in the configuration K_o , and at any t" (Teodosiu (1970)).

Therefore the transformation of N_X from K_o to K, characterizes the permanent deformation of N_X (according to (a),(i)).

Taking into account the above remarks, we shall call the local deformation E from K to k, the current elastic *deformation* and the local deformation P from K_o to K the *current plastic deformation*, respectively.[1]

Let us also observe that, since we accept the constitutive assumption that the rigid body rotations of the relaxed configurations can be distinct from one particle to the other, it is necessary to choose in a *rational* and not an arbitrary manner, the relaxed configurations in order to ensure the *continuity* of the mappings $X \in \mathcal{B} \rightarrow E(X,t)$, $P(X,t)$ for any moment t, with respect to $X \in \mathcal{B}$; this means a certain regularity with respect to the spatial variables. This requirement provides an additional argument for the the use of certain criteria in order to select the local relaxed configurations.

The use of l.c.r.i. configurations may ensure the required regularity.

The *multiplicative decomposition* $F = EPE_o^{-1}$ of the deformation gradient from k_o to k follows from the way in which E and P have been introduced (see Fig.1).

Fig.2 shows the essential distinction between the elastic and plastic deformations.

Fig.2.

In contrast with the elastic deformation the plastic permanent deformation does not change the mean orientation of the crystalline lattice and does not deform it.

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[1] Concerning the precise definitions of F,E and P we shall revert further on. The purpose of these considerations is the physical justification of the mathematical definitions, which will be introduced.

In particular, by pure elastic rotation the lattice is rotated together with N_X, by pure elastic strain the lattice is deformed together with N_X, by pure plastic rotation the material element is rotated, but in contrast with the case of pure elastic rotation, the lattice is not rotated together with N_X, by pure plastic deformation the material element is deformed, but in contrast with the case of pure elastic deformation the lattice is not deformed together with N_X (see Teodosiu (1970), Soós, Teodosiu (1983).)

Eckart (1948), Eglit (1960), Sedov (1962) were the first to use time-dependent, local, natural (relaxed) configurations, like K, in order to separate the elastic from the inelastic part of total deformation.

Lee, Liu (1967), (1968), Fox (1968 a,b) were the first to introduce independently the multiplicative decomposition rule for F in the special case $E_o=I$, I being the unit tensor.

Teodosiu (1970), Rice (1971) were the first to introduce independently the multiplicative decomposition rule in the general case $E_o \neq I$.

Teodosiu (1970), Rice (1971), Mandel (1971) for the first time and independently have considered the special choice of the orientation of the local relaxed configurations K.

1.3. The constitutive and evolution equations

An extensive literature exists concerning the constitutive theory of thermoelastoviscoplastic deformation of the materials with intermediate relaxed configurations and internal state variables. We shall limit ourselves to the main results obtained till 1983.

In order to describe the behaviour of materials, the authors use a thermoelastic constitutive equation $T=f(E,\theta,\alpha)$, based on the l.c.r.c, which is supplemented by the evolution equations for the plastic deformation and for the set α of the internal state variables.

Mandel (1971 d, Cap.3, §2) pointed out: "If hardening does not exists the state is the same in all succesive relaxed configurations. The cylindrical specimen can be very much plastically deformed and its actual state is independent on this stretching, depending on $E^T E$ and not on $F^T F$".

Lee and Liu (1968) emphasized: "The plastic deformation is given by change in permanent shape of the body from the initial configuration, considered to have uniform temperature, to the current state modified by removal of elastic strain and the thermal expansion by reducing the entire body again to the initial temperature".

The two authors underline: "In a problem, for which the elastic and plastic strains are both finite, the inclusion of the unstressed configuration is needed to achieve the simplification, concerning the use of unchanged thermo-elastic constants".

This fact, based on the experimental data, shows the importance of the using of the local relaxed configurations in order to introduce the elastic and plastic deformations. In this way, the use of the standard thermoelastic constitutive equations specifying the relation between the stress and the elastic deformation becomes possible. The fact that in spate of the plastic flow the "elastic constants based on the true stress and natural strain (determined by using the relaxed configurations) are effectively unchanged" (see Lee, Liu (1968)), reveals the advantage of utilizing the elastic deformation E determined via the plastically deformed configuration K.

Since "unloading a plastically deformed body will usually leave residual stress and to unstress all elements it would be necessary to dissect the body into infinitesimal volume elements, which will not fit together without the elastic deformations associated with the residual stress", Lee and Liu (1968) evidence that E and P do not represent the gradients of any global deformation of the body.

For this reason, in order to define rigorously the elastic and the plastic parts of the deformation, it is essential to use the concepts of local configuration and of local deformation, introduced by Noll (1967), (1968), (1972), (1973), (1974).

We insist on the fact that the model based on the local, current, relaxed, isoclinic configurations is justified by the experimental fact that "the effective uncoupling between elastic and plastic laws arises if the unstressed (permanently, plastically deformed) configuration is used as a reference state for deformation changes" (Lee, Germain(1974)).

Since it seems to us essential, we illustrate by one dimensional tests results the above general considerations (see also Haupt (1984)) of Lee, Liu (1967), (1968), Lee (1969), (1970), Lee, Germain (1974). In the case of the variant adopted by Lee, Liu (1967), (1968), Teodosiu (1970), Mandel (1971), (1972), the elastic deformation measured with respect to the l.c.r.i.c. is $\varepsilon^e=(l-l^p)/l^p$, where l and l^p are the final length and the length of the specimen determined after unloading, respectivelly, i.e. for the axial stress $\sigma=0$ and for $\theta=\theta_o$, when the deformation state is supposed to be homogeneous. In the Green, Naghdi (1965), (1968) considerations, the elastic deformation is referred to the initial configuration and it is expressed by $\bar{\varepsilon}^e=(l-l^p)/l_o$, where l_o is the initial length of the specimen. Since the total deformation is $\varepsilon=(l-l_o)/l_o$, in the first case we get $\varepsilon=\varepsilon^p+\varepsilon^e(1+\varepsilon^p)$ and in the second case $\varepsilon=\bar{\varepsilon}^e+\varepsilon^p$ where $\varepsilon^p=(l^p-l_o)/l_o$ represents the permanent, plastic deformation. Consequently, there exists the relationship $\bar{\varepsilon}^e=\varepsilon^e(1+\varepsilon^p)$ between the two "elastic" deformations $\bar{\varepsilon}^e$ and ε^e. We suppose that the elastic constitutive equation is linear and of the form $\sigma=C\varepsilon^e$ in the first case, and $\sigma=\bar{C}\bar{\varepsilon}^e$ in the second one. If C is <u>independent</u> on ε^p, according to experimental data, then it results that $\bar{C}$ depends <u>inevitably</u> on ε^p via the relation $\bar{C}=C/(1+\varepsilon^p)$.

The model considered here is useful in those cases in which the use of ε^e as a measure of elastic deformation leads to a very weak dependence (which can be neglected) of C on ε^p.

We consider that the dispute (see Lee, Germain (1974), Nemat-Nasser (1982), Lehmann (1982)) between the authors which made their option for one of these alternatives is laking any principled basis: the deformation (strain) is a purely geometric concept and the characterisation of one of its measure as elastic, is possible, justified and legitimate only in a constitutive framework. The criteria of utility, symplicity or rationality of the recalled type, which are motivated by experimental data must play a decissive role in the choosing the measure of elastic deformation.

Our point of view is clearly emphasized by the following remark made by Lee and Germain (1974): "This statement by Lee concerning the non-aditivity of elastic and plastic strains at finite deformation is misleading, since it was made on the assumption unstated that the plastic strain was expressed as permanent deformation from the undeformed state, and elastic strains deformation from the plastically deformed unstressed reference state, in order to achieve effective uncoupling of elastic and plastic properties. Additivity can be achieved at finite strain if such uncoupling is not demanded, as illustrated by Green, Naghdi's theory for instance".

It is clear that in the end only the experiment decides, if the use of ε^e instead of $\bar{\varepsilon}^e$ allows the uncoupling. The dislocations theory sugestes this alternative and speaks against $\bar{\varepsilon}^e$.

Mandel (1971d) furnished the conclusive arguments, based on the de dislocation theory, in favour of models using the l.c.r.i.c. He pointed out that the orientation of the current relaxed configuration is specified by the director frame, the existence and mechanical significance of which are given by (g), (h), (i). "A director frame is a frame associated with the atomic lattice (taken in a relaxed state), the elastic deformation is the deformation of crystalline lattice".

The following explanation of constitutive nature is essential to understand the model (Mandel (1971 d)): "Using those relaxed configurations for which the director frame (the crystalografic axes or the mean crystalografic axes) is keeping fixed the direction, the internal energy, entropy and free energy are functions of $(1/2)(E^TE-I)$, θ and $\{\alpha_j\}$ alone".

We point out that E is a local deformation from l.c.r.i.c. K to the actual configuration $\chi(.,t)$. Therefore, like Lee and Liu (1967), (1968), Mandel assumes that it is the use of K which ensures the explicit independence of the constitutive relations on the preceeding, permanent, plastic deformation P.

Like Teodosiu (1970), but in contrast with Lee and Liu (1967), (1968), Mandel shows that the use of l.c.r.i.c. leads to a model in which the role of the history of the total deformation (unknown in general) can be eliminated only if the internal state variables (in the Kröner's sense (1963)) are introduced. In order to specify the evolution of these quantities, Mandel states the valability of some evolution equations, but utilizing also the local, current, relexed, isoclinic configurations. At the same time, based on the microstructural considerations concerning the plastic deformation produced by the dislocation motion Teodosiu (1970), Mandel (1971,b,d), Kratochvil (1971) are the first to state that the evolution equation for the plastic deformation must be given for both the symmetric and antisymmetric parts of "the rate of plastic deformation", $\dot{P}P^{-1}$. [1)]

Taking into account the experimental data (with limited valability, depending on the values of the imposed deformations) Mandel in his theory assumes that any constitutive function has an invariant form (with respect to time) if it is relative to the l.c.r. isoclinic configuration. (This experimental evidence lies at the basis of our temporal invariance assumption.)

We point out that the existence of l.c.r.i. configurations, as well as the independence of the material response on the preceeding plastic deformation, (T depends on E - the elastic deformation) are constitutive hypothesis. Consequently, these assumptions determine the domain of the valability of the model (which must be experimentally determined) and the manner in which the elastic and plastic deformations are to be determined.

With regard to the first aspect, we can mentione

[1)] We shall analyse later the manner in which this requirement of Mandel has been implemented in the last years.

that (see Lee (1985)) "it has been found experimentally that in the moderate strain range (of about 30%) the usual elastic law which apply to an initially undeformed metal can be applied to the elastic deformation of the unstressed (current) plastically deformed configurations". Thus the model which will be presented can be applied to finite but moderate deformations.

Consequently, the model can be applied to materials with crystalline structure, because the hypotheses "are consistent with the physical basis of plastic flow since even after appreciable plastic strain only a small portion of atoms are disturbed from the regular atomic lattice and the atomic lattice determines the elastic constants of the material" (Lee, McMecking (1980)).

The limit of the applicability of the model results also from the following "hypothesis" made by Lee and McMecking (1980), and which are incorporated in our axiomatic system: "Definition of plastic flow as permanent deformation after all macroscopic stresses are removed provides a theory for materials which do not exhibit plastic flow on unloading". "In the unstressed configuration the stress have been removed and complete recovery of the elastic strain with no further plastic flow is considered to have taken place".

The constitutive functions involved are generally defined in the stress-temperature space. In order to reflect at macroscopic level the fact that the plastic deformation begins to develope only if the reduced tangential stress reaches a critical value (see (g)), we accept that the constitutive functions involved in the evolution equations depend on the stress state by means of the stress measures related to the l.c.r. configurations.

For this reason Teodosiu (1970), Kratochvil (1971), Mandel (1971), (1972), (1973) utilize as a measure for stress the second type Piola-Kirchhoff tensor $\Pi=(\det E)E^{-1}TE^{-T}$, with respect to the l.c.r.c., where T is the Cauchy stress.

Based on thermodynamic considerations, Teodosiu and Sidoroff (1975),(1986) propose the use of $\Sigma=(E^{T}E)\Pi$ which is a conjugate variable to $\dot{P}P^{-1}$.

Using a similar argument, Halphen and Nguyen (1975) propose the use of $R=(1/\bar{\rho})(E^{T}E)\Pi$, $\bar{\rho}$ being the mass density in the relaxed configuration, in contrast with Lee and Liu (1969), Lee and Mc.Mecking (1980), Lee and Lubarda (1981), Lee (1985) who introduce the Kirchhoff tensor $\tau = T(\det E)^{-1}$ as an independent variable in the evolution equations.

Although the yield (flow) conditions are generally given in the stress space, as a consequence of the equipresence principle, Kratochvil (1972) assumes that the constitutive functions involved in the evolution equations depend on the elastic deformation E.

Further on we do not refer any more to the theories that do not use the relaxed configurations in order to introduce the elastic and plastic deformations, like for instance those of Green, Naghdi (1965),(1968),Kratochvil, Dillon

(1969), (1970), Perzyna (1971),(1973),(1980).

Teodosiu (1970), Mandel (1971),(1972),(1973), Kratochvil (1972), Halphen (1975), Teodosiu and Sidoroff (1976) repeatedly stressed that at least in the case of anisotropic materials the rate of plastic deformation $\dot{P}P^{-1}$ itself, and not only its symmetric part, should be given by a constitutive law, as for instance the example of single crystals shows this in a very convincing way.[1)]

Since the dislocations play a principal role in producing the permanent plastic deformations, the most used internal state variables are those which specify in a more or less detalied way the dislocations distribution.

Obviously, in addition to the arguments given above one can introduce in a general theory other variables as well,which are more or less responsable for the plastic and/or viscoplastic deformation, for the isotropic or anisotropic hardening, and so on. In this sense, see for instance Teodosiu, Sidoroff (1976) who take account explicitly the influence of the evolution of the point defects, or Mandel (1971 d, Cap.3, §2), who state precisely, in an explicit way, the significance of internal state variables.

We consider that a well elaborated model must specify, in addition to the concrete nature of internal state variables, their transformation laws with respect to a change of frame, as well as, to the change of the reference configu-

[1)] For details see the references in Sidoroff and Teodosiu(1986).

rations. Obviously, these transformation laws are derived from the mechanical and physical significance of the employed state variables.

Although, it is well known that the constitutive functions depend essentially on the reference configurations used in order to formulate the material laws, this dependence is seldom mentioned in the papers which are founded on the use of the l.c.r.c. in order to introduce the elastic and plastic deformations. Neither are specified the transformation laws of the constitutive functions and of the internal state variables at the change of the current relaxed configurations.

Although the authors take into consideration tacitly these facts, the importance of these facts is explicitly pointed out in Mandel's papers elaborated by using consistently the director frame, through which is taken into account the specific role played in the theory by the crystalline structure of metals.

In the models based on the use of the time sequence of the l.c.r. configurations, the constitutive functions at any moment are referred to other configurations. Based on the physical properties (a),(c),(g),(h) one tacitly assumes that the functional form of the constitutive functions remain invariant in time, although the configurations used at each moment are different.

It is only Mandel who has pointed out the existence of this temporal invariance provided that the sequence of the l.c.r. configurations be an isocline set. Indeed, Mandel (1971d) has written: "Let f be a scalar function depending on the thermodynamic state (E,θ,α). If we consider the relaxed configurations for which the director frame D is fixed (i.e. the sequence of the isoclinic configurations) then at any time $f=\hat{f}(E^T E,\theta,\alpha)$ is a function of the thermodynamic variables in an <u>invariant</u> form".

Chap.2. Axiomatic reconstruction

In this section we deal with an axiomatic characterization of the thermoeleastoviscoplastic behaviour of metals. This reconstruction is based on the mechanical and physical results due to Lee, Liu (1967, 1968), Lee (1969, 1970), Teodosiu (1970, 1975), Sidoroff (1970, 1971, 1972, 1973, 1974), Teodosiu, Sidoroff (1976) on the one hand, and on the other hand, on the mathematical concepts due to Noll (1967, 1968, 1972, 1973, 1974).

The reconstruction takes into consideration the fact that the introduction of the local relaxed configuration becomes possible only if it is introduced simultaneously with the laws of material.

In the model, the history of deformation is involved by the actual value of the internal state variables only, or other hardening variables, which describe at the fenomenological level the essential features characterizing the configuration of the dislocations existing in the body.

In accordance with the characterization given by Kröner (1963) we understand that an external or internal (hidden) state variable is "any (macroscopic) quantity which can be measured at a given time without any information about the past". In this sense the elastic deformation, the yield stress, the temperature, the dislocation densities are state variables, but the existing plastic deformation has not this quality.

Since the elastic and plastic deformations are not the gradient of any global deformation of the body (see (i)) the axiomatic model will be developed by taking into account the local concepts elaborated by Noll.

Taking into consideration the physical origin and the mechanical significance of elastic and plastic deformations we shall use with consequence the sequence of the mobile <u>isoclinic</u> configurations.

The main idea lying at the basis of the axiomatic reconstruction <u>together</u> with the ideas of the above mentioned authors can be expressed as follows:

Although the elastic and plastic deformations are local deformations, they do <u>not</u> represent pure geometric or kinematic concepts. In order to characterize correctly these deformations it is necessary to employ the local current relaxed configurations which can be introduced only if we invoke constitutive and evolution equations.

This fact involves the consideration the dynamic concept of stress as well as the physical concept of internal state variables. For instance, an elastic eulerian fluid does not have a local relaxed configuration; it is clear that the assumption concerning the existence of such a configuration is a constitutive hypothesis, which must be explicitly formulated. It is also known that every symmetry group of crystalline fluids (see Coleman (1965), Wang (1965)) may contain a local nonorthogonal transformation. The exclusion of such a possibility in our model is also a constitutive assumption.

In conclusion: the consistent characterization of elastic and plastic deformations, hence of the thermoelastic-viscoplastic behaviour of a material, requires a simultaneous consideration of the geometric, kinematic, thermic and physic concepts taken together in the constitutive and evolution hypotheses of the model.

The axiomatic system presented subsequently represents an improved form of the one elaborated by Cleja-Ţigoiu (1988) and it is based on the earlier papers by Soós (1983) and Cleja-Ţigoiu (1983, 1984).

The properties derivated from the assumed axioms will be enounced without demonstration, but references will be given to the papers containing justifications of those results. Since all the considerations are local, relative to an arbitrary fixed material point X, the dependence on X of the various involved quantities will be understood and will not be explicitly mentioned whenever this does not produce confusions.

At the same time, for the sake of conciseness, we shall also omit the mentioning of the domains of definition of the involved constitutive functions.

In the following we denote by $E, V, L, L_i, L^+, S, S^+, O, O^+$ the three dimensional euclidian point space, the three dimensional real vector space (the translation space of E), the set of second order tensors, the subset of invertible tensors, the subset of second order tensors having positive determinant, the subset of symmetric tensors, the subset of symmetric and positive definite tensors, the subset of orthogonal tensors and the subset of proper orthogonal tensors, respectively. By R and R_+ we denote the set of real numbers and the subset of positive real numbers, respectively.

2.1. Material elements. Local configurations. Thermokinetic processes

In this section we present the mathematical concepts, developed by Noll (1967, 1968, 1972, 1974) which are necessary in order to define correctly the elastic and plastic deformations, which are local deformations.

We briefly recall the mathematical description for several physical concepts like body, material element, material neighbourhood of a given material points,... as given by Noll.

The body $\mathcal{B}$ is a set, whose members $X,Y,\ldots$ are called material points, endowed with a structure defined by a class $\mathcal{C}$ of mappings $k:\mathcal{B}\to\mathcal{E}$. The mappings $k\in\mathcal{C}$ are called the configurations of $\mathcal{B}$ (in the space $\mathcal{E}$). The spatial point $k(X)\in\mathcal{E}$ is called the place of the material points $X\in\mathcal{B}$ in the configuration k.

We say that $\mathcal{B}$ is a continuous body of class C^p $(p\geq 1)$ if the set $\mathcal{C}$ satisfies the following axioms:

C1) Every $k\in\mathcal{C}$ is one-to-one and its range $k(\mathcal{B})$ is an open subset of $\mathcal{E}$, which is called the region occupied by $\mathcal{B}$ in the configuration k.

C2) If $k,\bar{k}\in\mathcal{C}$ then $\lambda=\bar{k}\circ k^{-1}:k(\mathcal{B})\to\bar{k}(\mathcal{B})$ is a deformation of class C^p, which is called the deformation of $\mathcal{B}$ from the configuration k into the configuration $\bar{k}$.

C3) If $k\varepsilon\mathcal{C}$ and $\lambda:k(\mathcal{B})\to\mathcal{E}$ is a deformation of class C^p, then $\lambda\circ k\varepsilon\mathcal{C}$.

The mapping $\lambda\circ k$ is called the configuration obtained from the configuration k by the deformation λ.[1)]

Of central importance for further developements is the concept of local deformation at a material point X.

Two global configurations k and $\bar{k}$ are said to be equivalent at X and we write $k\sim_X\bar{k}$ if and only if $\nabla(\bar{k}\circ k^{-1})/_{k(X)}=I$.[2)] It is immediate that $\sim_X$ is an equivalence relation on $\mathcal{C}$. The resulting partition of $\mathcal{C}$ is denoted by $\mathcal{C}_X$, and its members K_X, $\bar{K}_X$,..., i.e. the equivalence classes, are called local configurations at X. Instead of writing $k\varepsilon K_X$ when k is an element of the class K_X we write $\nabla k(X)=K_X$, and we say that the local configuration K_X is the gradient at X of the global configuration k.

Let K_X, $\bar{K}_X\varepsilon\mathcal{C}_X$ be two local configurations and let $k\varepsilon K_X$, $\bar{k}\varepsilon\bar{K}_X$. The tensor $\nabla(\bar{k}\circ k^{-1})/_{k(X)}\varepsilon L_i$ depends only on K_X and $\bar{K}_X$ and not on the particular choices of $k\varepsilon K_X$ and $\bar{k}\varepsilon\bar{K}_X$. We denote this tensor by $\bar{K}_X K_X^{-1}$ and it is the local deformation from the local configuration K_X into the local configuration $\bar{K}_X$. Thus

$$\bar{K}_X K_X^{-1}\equiv\nabla(\bar{k}\circ k^{-1})/_{k(X)}=\nabla\bar{k}(X)(\nabla k(X))^{-1}.$$

1) It follows that $\nabla\lambda(X)\varepsilon L_i$.

2) I represents the unit tensor

It follows that any local deformation belongs to L_i and conversely, any element from L_i is a local deformation.

If $K_X \varepsilon C_X$ and F is any local deformation, $F \varepsilon L_i$, we can define a new local deformation FK_X by

$$FK_X \equiv \{\lambda \circ k \,|\, \nabla\lambda|_{k(X)} = F, \quad \nabla k(X) = K_X\},$$

FK_X is called the <u>local configuration obtained from the local configuration</u> K_X <u>by the local deformation</u> F.

The following relationships take place

$$(\bar{K}_X K_X^{-1}) K_X = \bar{K}_X \;, \quad (FK_X) K_X^{-1} = F.$$

Using the above results we define the tangent space as follows:

Consider the set $\{(K_X, u)/K_X \varepsilon C_X, \; u \varepsilon V\} \equiv C_X x V$. We say that two pairs (K_X, u) and $(\bar{K}_X, \bar{u})$ <u>are equivalent</u> if $(\bar{K}_X K_X^{-1}) u = \bar{u}$.

It follows that the above relation defines an equivalence relation. The resulting equivalence classes are called tangent vectors at X and are denoted by $u_X, \ldots$. The totality of all these tangent vectors is denoted by T_X and it is called the tangent space at $X \varepsilon B$.

Let $K_X \varepsilon C_X$; we say that $u_X \varepsilon T_X$ determines a unique spatial vector $u \varepsilon V$ such that $(K_X, u) \varepsilon u_X$. We can therefore use the notation

$$u=K_X u_X, \quad u_X=K_X^{-1} u \quad \text{if} \quad (K_X, u) \varepsilon u_X,$$

and we say that K_X determines a one-to-one mapping of the tangent space T_X onto the space V. The tangent space T_X has the natural structure of three-dimensional vector space, with the addition defined by

$$u_X+v_X=K_X^{-1}(u+v) \quad \text{if} \quad u_X=K_X^{-1}u, \quad v_X=K_X^{-1}v,$$

and the multiplication with scalars by

$$au_X=K_X^{-1}(au) \quad \text{if} \quad u_X=K_X^{-1}u, \ a \varepsilon R.$$

It is straightforward that u_X+v_X and au_X are well defined because the results are independent of the choice of the local configuration K_X used to represent u_X and v_X in V.

We remark that the local configurations can be identified with the invertible linear transformation of T_X onto V.

The tangent vectors and the tangent space at the material point X are the intrinsec mathematical models of the physical concepts of material elements at X and of the infinitesimal material neighbourhood of a given point X, respectively.

Let X be a fixed material point.

We consider the set of all pairs (χ,θ) with χ the motion and θ the temperature, defined on $N_X xR$, where N_X is a certain neighbourhood of the material point X, $N_X \subset B$. (χ,θ) is called the (local) thermokinetic process of N_X.

We recall that by definition a motion χ of the body B is a one-parameter family of configurations of B and moreover for all $Y \varepsilon N_X$, $\chi(Y,\cdot):R \rightarrow E$ is continuously differentiable function of class C^2. The thermokinetic processes have certain properties listed in Coleman (1964).

In the following we denote by (χ^*,θ^*) the process obtained from (χ,θ) by a change of frame defined by the orthogonal time-dependent tensor Q; all quantities associated with (χ^*,θ^*) will be denoted by a superposed $*$.

2.2. Thermoelastic constitutive equations

We say that a continuous body B is made of a thermoelastoviscoplastic (t.e.v.p.) material, if the following constitutive assumptions are met for all material points $X \varepsilon B$:

A.1. For all X and all $t \varepsilon R$, for any admissible thermokinetic process (χ,θ) there exist K_t, called local current configuration (l.c.c.), together with the set of internal state variables (i.s.v.) $\{\alpha_{K_t}\}$ restricted by the requirements listed under the form of several axioms.

D.1. The local current thermoelastic deformation E and the local current thermoplastic deformation P are defined by

$$E=\nabla\chi_t K_t^{-1}, \quad P=K_t K_o^{-1} \tag{1}$$

In order to simplify the presentation, we assume the content of dislocations in the reference configuration k to be negligible, and consequently $K_o=\nabla k$.

From (1) we obtain (in contrast with Clifton (1972)), the noncommutative multiplicative decomposition of the deformation gradient $F=\nabla\chi_t\nabla k^{-1}=\nabla\chi_t K_o^{-1}$:

$$F = EP \tag{2}$$

With respect to the local current configuration $\nabla\chi_t$, we denote by T,ρ,q and g the Cauchy stress tensor, the mass density, the heat flux vector and the temperature gradient, respectively. Consequently, the corresponding quantities with respect to the local current configuration K_t can be introduced by

$$\begin{aligned} &\pi=\pi_{K_t}=\det E\ E^{-1}TE^{-T},\ \bar{\rho}=\rho_{K_t}=\rho\ \det E, \\ &\bar{q}=q_{K_t}=\det E\ E^{-1}q,\ \bar{g}=g_{K_t}=E^{T}g. \end{aligned} \tag{3}$$

A.2 (The existence of the constitutive equation of the thermoelastic type). For any (χ,θ) the specific free energy ψ, the specific entropy η, the Cauchy stress T and the heat flux vector q are determined by the following constitutive relations:

$$\psi=\psi_{K_t}(\Lambda,\alpha_{K_t}),\ \eta=\eta_{K_t}(\Lambda,\alpha_{K_t}),$$
$$T=f_{K_t}(\Lambda,\alpha_{K_t}),\ q=p_{K_t}(\Lambda,g,\alpha_{K_t}). \tag{4}$$

where $\Lambda=(E,\theta)$ and $g=\nabla\theta$[1].

The constitutive functions from (4) are related to the local current configuration K_t.

P.1. If K_t and $\overline{K}_t$ are the l.c.c. corresponding to the same process (χ,θ) then

$$f_{\overline{K}_t}(\overline{E},\theta,\alpha_{\overline{K}_t})=f_{K_t}(\overline{E}G_t,\theta,\alpha_{K_t}) \tag{5}$$

where $\overline{E}=\nabla\chi_t\overline{K}_t^{-1}$, $G_t=\overline{K}_tK_t^{-1}$, and $\alpha_{\overline{K}_t}$, α_{K_t} are related by the transformation G_t, depending on the physical significance attached to the internal variables.

A.3 (Objectivity assumption). If (K_t,α_{K_t}) is a pair of l.c.c. and of i.s.v. associated to the process (χ,θ), then

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1) For the application to physical situation it is necessary to limit the domain of the material functions. For brevity we do not supply the mathematical details which may arise in the consideration of limitations of this kind.

K_t is a l.c.c. for the process (χ^{*},θ^{*}) as well, and $\alpha^{*}_{K_t}$ - the set of i.s.v. corresponding to the process (χ^{*},θ^{*}), but related to K_t -, is equal to α_{K_t}; i.e. $\alpha^{*}_{K_t}=\alpha_{K_t}$.

P.2. If we suppose that $\psi^{*}=\psi$, $\eta^{*}=\eta$, $T^{*}=QTQ^{T}$, $q^{*}=Qq$, then from A.2 and A.3 it results

$$P^{*}=P,\ E^{*}=QE,\ \pi^{*}=\pi,\ \rho^{*}_{K_t}=\rho_{K_t},\ g^{*}_{K_t}=g_{K_t},\ q^{*}_{K_t}=q_{K_t} \tag{6}$$

and for the thermoelastic constitutive relations we obtain the following reduced form

$$\begin{aligned}&\psi=\bar{\psi}_{K_t}(\Lambda^{e},\alpha_{K_t}),\ \eta=\bar{\eta}_{K_t}(\Lambda^{e},\alpha_{K_t}),\pi=h_{K_t}(\Lambda^{e},\alpha_{K_t}),\\ &q_{K_t}=\bar{p}_{K_t}(\Lambda^{e},g_{K_t},\alpha_{K_t})\end{aligned} \tag{7}$$

where

$$\Lambda^{e}=(C^{e},\theta) \quad \text{with} \quad C^{e}=E^{T}E\ . \tag{8}$$

A.4 (The relaxation assumption). The thermoelastic constitutive function f_{K_t} and the configuration K_t satisfy the relation

$$f_{K_t}(I,\theta_o,\alpha_{K_t}) = 0 \tag{9}$$

where θ_o is the initial temperature. Moreover, if $f_{K_t}(S,\theta_o,\alpha_{K_t})=0$, for S-a symmetric and positive defined tensor -, then

S=I.

On the basis of the property stated in A.4 the configuration K_t will be called the local current relaxed configuration (l.c.r.c.).

As a consequence of A.3, A.4 and P.1 it follows

P.3. Let K_t and $\overline{K}_t$ be defined as in P.1.

If K_t is a l.c.r.c. then $\overline{K}_t$ is also a local current relaxed configuration and G_t is an orthogonal transformation.

In particular, it results that $\overline{K}_o = QK_o$ with $Q \in \mathcal{O}$.

§3.3. The evolution equations.

A.5 (Evolution equations). The evolution in time of the pair (K_t, α_{K_t}) is given by the relations

$$\begin{aligned} \dot{P}P^{-1} &= A_{K_t}(\xi, \alpha_{K_t}) + \langle\lambda\rangle B_{K_t}(\xi, \alpha_K t) \\ \dot{\alpha}_{K_t} &= \ell_{K_t}(\xi, \alpha_{K_t}) + \langle\lambda\rangle m_{K_t}(\xi, \alpha_{K_t}) \end{aligned} \qquad (10)$$

with the initial data $P(0)=I$, $\alpha_{K_o}(0)=\alpha_o$, where $\xi=(\pi,\theta)$.

A.6. The material functions from (10) have the following properties:

$$B_{K_t}(\xi, \alpha_{K_t})=0,\ m_{K_t}(\xi, \alpha_{K_t})=0 \text{ if } \mathcal{F}_{K_t}(\xi, \alpha_{K_t})<0,$$

$$A_{K_t}(\xi, \alpha_{K_t})=0 \quad \text{if } \mathcal{G}(\xi, \alpha_{K_t})\leq 0,$$

where G_{K_t} -the viscoplastic function and F_{K_t} - the plastic function satisfy the conditions:

$$G_{K_t}(\xi_o,\alpha_{K_t})<0,\ F_{K_t}(\xi_o,\alpha_{K_t})<0 \quad \text{for } \xi_o=(0,\theta_o).$$

The plastic loading factor λ is defined by

$$\lambda=\partial_\xi F(\xi,\alpha_{K_t}).\dot{\xi}^+ \quad \text{with} \quad \dot{\xi}^+=(\dot{\pi}^+,\dot{\theta}^+)$$

for any ξ lying on the current yield surface (also called the instantaneous plasticity surface)

$$S(t) = \{\xi |\ F_{K_t}(\xi,\alpha_{K_t})=0\}.$$

The material functions involved in (10) also satisfy the consistency condition

$$1+\partial_\alpha F_{K_t}.m_{K_t}=0,\quad \partial_\alpha F_{K_t}.\ell_{K_t}=0 \quad \text{on} \quad S(t). \tag{11}$$

A.7. (Assumption of temporal invariance). The configurations K_t and the set of material functions satisfy the condition[1]:

$$f_{K_t}(\Lambda,\alpha)=f_{K_{t'}}(\Lambda,\alpha),\ldots,\ F_{K_t}(\xi,\alpha)=F_{K_{t'}}(\xi,\alpha)$$

$t,t'\in R.$

[1] Further on, whenever misinterpretation is unlikely the explicit mentioning of the dependence of the material functions on the l.c.r.i.c. will be omitted.

The configurations K_t satisfing A.1-A.7 will be called local current relaxed isoclinic configuration (l.c.r.i.c.).

Obviously, A.7 excludes the explicit dependence of the material response on the accumulated plastic deformations.

A.8. (Causality assumption)[1]. For all thermokinetic process (χ,θ) and at any time t, the loading plastic factor λ is uniquelly determined if the current values of (π,θ) are on the current yield surface.

From the relations (2), (7), (8), A.7 and A.8 results the following proposition asserting the existence of the complementary plastic factor β and of the hardening modulus γ:

P.4. (Cleja-Ţigoiu (1983)). Let (χ,θ) be given.

The functions β and γ exist and are defined by

$$\text{a)}\ \beta = P^{-1}\partial_{C^e}\tilde{F}(\zeta)P^{-T}.\overset{\bullet +}{C} + \partial_{\Theta}\tilde{F}(\zeta).\overset{\bullet +}{\theta} + \partial_{\alpha}\tilde{F}(\zeta).\tilde{\ell}(\zeta) - -2\partial_{C^e}\tilde{F}(\zeta).\{C^e A(\zeta)\}^s \tag{12}$$

and

$$\gamma = 2\partial_{C^e}\tilde{F}(\zeta).\{C^e\tilde{B}(\zeta)\}^s - \partial_{\alpha}\tilde{F}(\zeta).\tilde{m}(\zeta)$$

for all ζ such that $\tilde{F}(\zeta)=0$.

b) β and λ have the same signum, $\gamma>0$ and $\langle\lambda\rangle = \gamma^{-1}\langle\beta\rangle$.

We recall that in (12)

1)See also Sidoroff (1974) and Halphen (1975).

$$\tilde{F}(\zeta)=F(h(C^e,\theta,\alpha),\theta,\alpha) \quad \text{with } \zeta=(C^e,\theta,\alpha) \qquad (13)$$

and

$$C = F^T F = P^T C^e P$$

Subsequently, we denote by $\sim$, superposed on the symbol of the considered function, the function obtained by the procedure indicated in (13).

In the constitutive framework accepted here, assuming that Clausius-Duhem inequality represents a restriction on the admissible thermodynamic processes and using the complementary plastic factor β, the procedure employed by Coleman and Gurtin (1967) can be used in order to obtain thermodynamic constitutive restrictions.

Consequently, we have (for details see Cleja-Ţigoiu (1983):

P.5. i) The constitutive function $\overline{\psi}$ is a thermodynamic potential, i.e.

$$\pi=2\overline{\rho}\,\partial_{C^e}\overline{\psi}(\zeta), \quad \eta=-\partial_\theta\overline{\psi},$$

ii) $\overline{\psi}$, $\overline{p}$ and the viscoplastic functions $\tilde{A}$ and $\tilde{l}$ satisfy the dissipation inequality

$$2\partial_{C^e}\overline{\Psi}(\zeta).\{C^e\tilde{A}(\zeta)\}^s-\partial_\alpha\overline{\Psi}(\xi).\tilde{\ell}(\xi)-\frac{1}{\Theta\overline{\rho}}\overline{g}.\overline{p}\geq 0.$$

iii) For all ζ such that $\tilde{F}(\zeta)=0$, regular points for the yield surface in the strain-temperature space, the "energy" equality holds:

$$\ell\equiv 2\,_{C^e}\overline{\Psi}(\zeta).\{C^e\tilde{B}(\zeta)\}^s-\partial_\alpha\overline{\Psi}(\zeta).\tilde{m}(\zeta)=0.$$

§2.3. Material symmetry

A.9. For any orthogonal transformation $Q\varepsilon\mathcal{O}$ and for any set of l.c.r.i.c. $I_{K_o}=\{K_t|t\varepsilon R_+\}$ corresponding to a thermokinetic process (χ,Θ), the set $I_{\overline{K}_o}=\{\overline{K}_t=QK_t|K_t\varepsilon I_{K_o},\ t\varepsilon R_+\}$ is a set of l.c.r.i.c. corresponding to the same (χ,Θ) and to the local initial relaxed configuration $\overline{K}_o=QK_o$.

Conversely: if $I_{K_o}=\{K_t|t\varepsilon R_+\}$ and $I_{\overline{K}_o}=\{\overline{K}_t|t\varepsilon R_+\}$ are two sets of l.c.r.i.c. associated to the same process (χ,Θ) then a local orthogonal deformation Q exists such that $\overline{K}_t=QK_t$ for all $t\varepsilon R_+$.

Assuming that A.9 holds, it follows:

P.6. The quantities corresponding to the same thermokinetic process (χ,Θ) but refered to the sets of the l.c.r.i.c. I_{K_o} and $I_{\overline{K}_o}$, respectively, are related by

$$\overline{E}=EQ^T,\ \overline{C}^e=QC^eQ^T,\ \overline{\pi}=Q\pi Q^T,\ \overline{P}=QPQ^T, \tag{14}$$

$$\rho_{\overline{K}_t}=\rho_{K_t}.$$

The internal variables α_{K_t} and $\alpha_{\overline{K}_t}$ considered with respect to the above mentioned sets of l.c.r.i.c. are related through Q, depending on the attached physical significance and must be explicitely stipulated, taking into account the entities that are to be modelled.

We simbolically denote this link by

$$\alpha_{\overline{K}_t} = Q[\alpha_{K_t}] \tag{15}$$

For instance, if α_{K_t} represents the density of the dislocation line per unit volume in the l.c.r.i.c. then $\alpha_{K_t}=\alpha_{\overline{K}_t}$. When α_{K_t} represents the Noll's type density of dislocation (1967, 1968) or when $\bar{\alpha}_{K_t}$ is the Piola-Kirchhoff type of back stress (with respect to the l.c.r.i.c.) then

$$\alpha_{\overline{K}_t}=Q\alpha_{K_t}Q^T.$$

If we take into account A.7, from (14) used in the constitutive and in the evolution equations, the following property results:

<u>P.5.</u> The material functions with respect to two sets of l.c.r.i.c. generated by the local initial configurations K_o and $\overline{K}_o=QK_o$, respectively, are related by

$$f_{\overline{K}_t}(E,\theta,\alpha)=f_{K_t}(EQ,\theta,Q[\alpha])$$

$$h_{\overline{K}_t}(QCQ^T,\Theta,Q[\alpha])=Qh_{K_t}(C,\Theta,\alpha)Q^T$$
$$\ell_{\overline{K}_t}(Q\pi Q^T,\Theta,Q[\alpha])=Q[\ell_{K_t}(\pi,\Theta,\alpha)] \qquad (16)$$
$$F_{\overline{K}_t}(Q\pi Q^T,\Theta,Q[\alpha])=F_{K_t}(\pi,\Theta,\alpha)$$

The material functions A,B obey the same relations like h, but G,ψ,η transform under the change of mentioned l.c.r.i.c. like F, while m satisfies the same relation as ℓ. Finally, we have

$$\overline{p}_{\overline{K}_t}(QCQ^T,\Theta,Qg,Q[\alpha])=Q\overline{p}_{K_t}(C,\Theta,g,\alpha)$$

In the previous relations $E\varepsilon L_i$, $C\varepsilon S^+$, $\pi\varepsilon S$, $g\varepsilon V$.

On the basis of P.5 we addopt the following definition <u>for the material symmetry</u>:

<u>D.2</u>. The configurations $K_t\varepsilon I_{K_o}$ and $\overline{K}_t\varepsilon I_{\overline{K}_o}$, with $\overline{K}_t=$ $=QK_t$ and $Q\varepsilon O$, are called <u>thermoelastoviscoplastic equivalent</u> if the material functions corresponding to K_t satisfy the restrictions:

$$f_{K_t}(E,\Theta,\alpha)=f_{K_t}(EQ,\Theta,Q[\alpha]),$$

or

$$h_{K_t}(QCQ^T,\Theta,Q[\alpha])=Qh_{K_t}(C,\Theta,\alpha)Q^T,$$
$$\ell_{K_t}(Q\pi Q^T,\pi,Q[\alpha])=Q[\ell_{K_t}(\pi,\Theta,\alpha)],$$

$$\mathcal{F}_{K_t}(Q\pi Q^T,\theta,Q[\alpha])=\mathcal{F}_{K_t}(\pi,\theta,\alpha).$$

The material functions A,B satisfy the same relations like h; G,ψ,η like $\mathcal{F}$, while

$$\bar{p}_{K_t}(QCQ^T,\theta,Qg,Q[\alpha])=Q\bar{p}_{K_t}(C,\theta,g,\alpha).$$

The transformation Q is called the symmetry transformation corresponding to the thermoelastoviscoplastic response and to the local configuration K_t.

We denote by g_{K_t} the set of all orthogonal symmetry transformations which obey D.2. Taking also into account the temporal-invariance assumption the proposition below is straightforward.

P.6. g_{K_t} is a subgroup of the group of orthogonal transformations $\mathcal{O}$ and $g_{K_t}=g_{K_o}$ for all $K_t\varepsilon I_{K_o}$.

We call this group the material symmetry group corresponding to the set of l.c.r.i.c., I_{K_o}.

For a given (χ,θ) on the basis of the material symmetry, by taking into account the constitutive and evolution equations, the following non-uniqueness of elastic and, plastic deformations results, even if K_o is fixed and consequently A.9 holds.

P.7. Let Q be in g_{K_o}.

1) If (P,α) is a solution for the evolution equations with <u>the initial data</u> $P(0)=I$, $\alpha(0)=\alpha_o$, then $\hat{P}=QP$, $\hat{\alpha}=Q[\alpha]$ is a solution for the same evolution equations but with the <u>changed</u> initial conditions $\hat{P}(0)=Q$, $\hat{\alpha}(0)=Q[\alpha_o]$.

2) Moreover, we have $F=\hat{E}\hat{P}=EP$ with $\hat{E}=EQ^T$ and

$$T=f_{K_o}(E,\theta,\alpha)=f_{K_o}(\hat{E},\theta,\hat{\alpha}).$$

2.4. Commentaries to A.1-A.9

1. The statements made in A.3 expresses the following property of mechanical significance: the current, relaxed, only plastically deformed configurations and the internal variables with respect to these configurations are invariant under a change of frame, hence they are invariant under a rigid motion superposed on the given thermokinetic process. The formulae (6) which are consequences of A.3. were utilized without any explicit justification by Teodosiu (1970), Kratochwil (1971, 1972) and others.

2. The definition of the plastic deformation given by $(1)_2$, together with the relaxation axiom A.4, are based on the fact that the plastic remanent deformation is associated with an unstressed state of a small neighbourhood of the particle.

3. A local current relaxed configuration K_t represents the mathematical model describing the unstressed perfect crystalline lattice of the neighbourhood of the particle from the crystalline or polycrystalline metal.

4. The elastic deformation E, defined with the help of the local current relaxed isoclinic configuration $K_t \varepsilon I_{K_o}$, is a measure of the deformation of the crystalline lattice, the relaxed position of which was fixed for all moments t, by choosing (to a great extent arbitrarily) the initial local relaxed configuration K_o.

5. From the last two remarks it follows that E characterizes that part of the deformation of the body which is reversible.

6. The first part of A.4. emphasizes precisely the reversible nature of the pure elastic deformation: this means that (at least locally) there exists an one-to-one correspondence between the stress state and the pure elastic deformations state in a neighbourhood of the given particle.

7. From the multiplicative decomposition of F we obtain that the velocity gradient is expressed by

$$L=\dot{E}E^{-1}+E\,\dot{P}P^{-1}E^{-1}.$$

According to the accepted interpretations $L^e=\dot{E}E^{-1}$ represents the rate of elastic deformation and $L^p=\dot{P}P^{-1}$ the rate of plastic deformation. Consequently $D^e=\{\dot{E}E^{-1}\}^s$ and $W^e=\{\dot{E}E^{-1}\}^a$ represent the elastic rate of strain and the elastic spin, respectively. On the other hand $D^p=\{\dot{P}P^{-1}\}^s$ and $W^p=\{\dot{P}P^{-1}\}^a$ represent the plastic rate of strain and the plastic spin, respectively. The evolution equations prescribed for D^p and W^p have a mechanical meaning.

8. The evolution equations introduced by A.5 are written in an invariant form under the change of frame. Consequently, the isotropy of the physical space does not impose any additional restrictions on the material functions presented in (10).

9. The plastic loading factor is defined in a classical maner (as in Green, Naghdi (1968), Teodosiu (1970), Sidoroff (1972)) in contrast to Mandel (1972, 1973) so that the loading ($\lambda>0$) (hence a variation of the plastic deformation), the unloading and the neutral process, respectively, is established according to the direction taken by $\dot{\xi}^{+}\varepsilon S\times R$ in the stress-temperature space, either outwards, or inwards, or finally tangent to the yield surface (when this is a closed surface).

10. The evolution equations (10) contain both viscoplastic and instantaneous plastic terms in order to describe the viscoplastic effects, as well. These can occour during unloading characterized by $\lambda<0$ (see Mandel (1971, 1972, 1973) and Halphen (1975)).

11. The restrictions imposed to the material functions from (10) by A.6 take into account the fact that the change of the internal variables can occour also during the processes in which the plastic deformations remain unchanged in time. This property is shown by experimental data for hot deformed metals, for instance.

12. The causality assumption A.8 ensures the consistency of A.1 and A.6, because it allows the introduction of

the complementary plastic factor β, with which we can determine the evolution of (P,α) based on the knowledge of the history of the thermokinetic process (χ,θ).

Indeed, the knowledge of λ requires the knowledge of the history of π which, in its turn, depends on the history of P and thus generally λ can not be determined when the elastic deformations are large, even if the history of T is given.

13. Further one usually considers (see Nguyen, Halphen (1975), Mandel (1971)), that the constitutive restrictions obtained in the viscoplastic case still remain valid when we take into consideration the instantaneous plastic terms as well. These restrictions can be found correctly only by using the complementary plastic factor, the existence of which is assured by the axiom A.8 (for the proof see Sanda Cleja-Ţigoiu (1988)).

14. The statement contained in A.7 is tacitly admitted in all the papers which use the model analysed here. The temporal invariance condition stipulated in A.7. is based on the physical properties h) and i) mentioned in §1.1.At the same time, the property emphasized in A.7 points out the advantages, as well as the limitatiens of the model based on the l.c.r.i.c.

15. In connection with the preceding remark we mention that A.9. underlines the fact that any thermoelastic deformation is the deformation of the unstressed perfect crystalline lattice. At the same time, A.9 shows that any thermoplastic deformation[1] leaves the crystalline structure unchanged, its irrever-

1) Within the limitatiens of the model's applicability.

sible, permanent nature is thus ensured. The appearence of the plastic deformation is due mainly to the motion, generation and rearrangement of the dislocations.

16. For a given thermokinetic process both types of deformations can be related to the arbitrary local current relaxed reference configurations, but these configurations must be the elements of a certain set of l.c.r. isoclinic configurations. Such kind of configuration sets can be obtained from the perfect unstressed crystalline lattice via a rigid motion remaining the same at any moment t. The degree of arbitrariness is reduced to the choosing of the local relaxed isoclinic configuration at the initial moment.

17. While the initial values α_o of the internal state variables (in the Kröner's sense) can be determined, the initial value of the permanent plastic deformation (which is not an internal state variable) is in principle unknown and practically undeterminable. The definition of the plastic deformation is consistent with the mentioned facts and leads automatically condition $P(0)=I$.

Fig.3

18. The deformation $\hat{P}$ from P.7. does not correspond to an isoclinic set of l.c.r.c. and hence it is not a plastic deformation in the sense of the definition given here. On the contrary, $\hat{E}$ from P.7. corresponds to an isoclinic set of l.c.r.c. $I_{\overline{K}_o}=I_{QK_o}$ and, therefore, it represents an elastic deformation, according to our definition. The relationship between $(\hat{P}, \hat{E})$ and $(\overline{P}, \overline{E})$ is given by $\hat{P}=\overline{P}Q$, $\hat{E}=\overline{E}$ (see Fig.3). From these relations it follows that "the plastic deformation"

$\hat{P}$ represents an elastic rigid rotation of K_o in the equivalent configuration $\overline{K}_o$ followed by the plastic deformation $\overline{P}$ of $\overline{K}_o$. Obviously, we have $T = f_{K_o}(E,\theta,\alpha_{K_t}) = f_{K_o}(EQ^T,\theta,Q^T[\alpha_{K_t}])$, which justifies the interpretation given above.

According to the above arguments we can state that there are several elastic and "plastic" deformations, which correspond to a given motion and to a fixed K_o. Thus one can not distinguish between two equivalent sets of l.c.r.i.c. on the basis of the thermoelastoviscoplastic response of the material.

19. At the same time, we remark that from the polar decomposition $P=R^pU^p$ and $\hat{P}=\hat{R}^pU^p$ it results $\hat{U}^p=U^p$ and from $E=V^eR^e$ and $\hat{E}=\hat{V}^e\hat{R}^e$ it follows $\hat{V}^e=V^e$ (there $R^p,\hat{R}^p,\hat{R}^e,R^e \in \mathcal{O}^+; U^p,V^e,\hat{V}^e \in \mathcal{S}^+$). Consequently, once K_o is fixed the "pure" plastic strain U^p and the pure elastic deformation V^e are uniquelly determined even when we replace K_o by an equivalent configuration $\overline{K}_o=QK_o$. Consequently, it results from here that, the Lee's choice according to which the elastic rotations are equal to I, during unloading, can be applied for the isotropic bodies only, i.e. $g_{K_o}=\mathcal{O}$.

Chap.3. Plastic rotation. Anisotropic hardening

Concerning the formulation of the constitutive equations in the early eithies, Dafalias (1983) mention:

"The definition of the proper co-rotational rates for the stress and internal structure variables is still being debated in the macroscopic constitutive formulation at large deformation elastoplasticity: In his elucidative work Mandel (1971d)(so as Kröner, Teodosiu (1970, 1972), Kratochvil (1971, 1972a,b, 1974), our note) supplemented the notion of the multiplicative decomposition of the deformation gradient into an elastoc and plastic part, introduced by Lee, Liu (1967, 1968) (and Lee, Liu (1969, 1970), Fox (1968a,b), Kröner, Teodosiu (1970, 1972), Mandel (1971a,b,c,d), Kratochvil (1971, 1972a,b, 1974), our note) with director vectors attached to the material substructure in the relaxed configuration. Motivated by well understood concepts of the micromechanics of crystalline structures, Mandel defined as proper corotational rate the one associated with the spin of the triad of the director vectors. However, he never tries to present a systematic macroscopic derivation of analytical expression for this spin and instead, he attempted to approach the problem from microstructural consideration (also see Teodosiu, Sidoroff (1976)), a formidable task not yet fully developed".

The main aim of the papers elaborated soon after 1980 was to provide the "missing link" for the macroscopic formulation and the application of Teodosiu (1970, 1975), Lee, Liu (1969, 1970), Mandel (1971, 1972, 1973) theories, to extend and generalize certain of its aspects and to illustrate it by examples. An immediate consequence of these theories was that evolution equations were required not only for the plastic rate of strain $\{\dot{P}P^{-1}\}^s$, but also for the plastic spin $\{\dot{P}P^{-1}\}^a$. The plastic spin was introduced also since the evolution of the stress state is determined only if the evolution of the elastic deformation $E=FP^{-1}$ is known, and consequently, the evolution of P is also necessary.

3.1. Rotation of the direction frame

In order to formulate the material laws, Mandel (1971 d, chap.I, §4.2) used, as a first step, a set of local current relaxed isoclinic configuration or, using his therminology "the configuration is relaxed so that the direction frame keeps fixed its orientation".

Starting with Mandel's fundamental idea, we were lead to the formulation of the constitutive equations by the complete set of the axioms A.1-A.9. We give precise formulation of both the assumptions clearly stated by Mandel, and of the hypotheses only tacitly accepted and currently used by him in his subsequent developments.

Assuming tacitly the valability of P.3 Mandel has formulated the laws of the material with respect to the arbitrarily relaxed configurations $\overline{K}_t$ obtained from the isoclinic ones by an orthogonal transformation, denoted by β_t.

Let be $\overline{K}_t=\beta_t K_t$ with $K_t \in I_{K_o}$ and $\overline{P}=\overline{K}_t K_o^{-1}$; it follows that

$$\bar{P}=\beta_t P,\ F=\bar{E}\bar{P},\ \bar{E}=\nabla\chi_t \bar{K}_t^{-1}=E\beta_t^T,\ \bar{\pi}=\det\bar{E}\ \bar{E}^{-1}T\bar{E}^{-T}=$$
$$=\beta_t\pi\beta_t^T,\ \alpha_{\bar{K}_t}=\beta_t[\alpha_{K_t}].$$

The following proposition[1)] is a consequence of the above relations

<u>P.8</u>. If A.1-A.9 hold then the following relations take place:

$$T=f_{\bar{K}_t}(\bar{E},\Theta,\alpha_{\bar{K}_t},\beta_t)\equiv f_{K_t}(\bar{E}\beta_t,\Theta,\beta_t^T[\alpha_{\bar{K}_t}])$$

and $f_{\bar{K}_t}(I,\Theta_o,\alpha)=0$,

$$\frac{D\bar{P}}{Dt}\bar{P}^{-1}=A_{\bar{K}_t}(\bar{\pi},\Theta,\alpha_{\bar{K}_t},\beta_t)\equiv\beta_t A_{K_t}(\beta_t^T\bar{\pi}\beta_t,\Theta,\ \beta_t^T[\alpha_{\bar{K}_t}])\beta_t^T\ ,$$

$$\frac{D\alpha_{\bar{K}_t}}{Dt}=\ell_{\bar{K}_t}(\bar{\pi},\Theta,\alpha_{\bar{K}_t},\beta_t)\equiv\beta_t[\ell_{K_t}(\beta_t^T\pi\beta_t,\Theta,\beta_t^T[\alpha_{\bar{K}_t}])$$

where

$$\frac{D\bar{P}}{Dt}=\dot{\bar{P}}-\omega\bar{P},\quad \omega=\dot{\beta}_t\beta_t^T$$

and the formula for $\frac{D\alpha_{\bar{K}_t}}{Dt}$ depends on the concrete form of the relationship between $\alpha_{\bar{K}_t}$ and α_{K_t}. For instance, if α_{K_t} represents the density of dislocation line then we have

1) In order to simplify the presentation we have considered here the case without instantaneous plasticity only.

$\frac{D\alpha_{\overline{K}_t}}{Dt} = \dot{\alpha}_{\overline{K}_t}$, or if $\alpha_{\overline{K}_t}$ represents the Piola-Kirchhoff type of back stress tensor relative to $\overline{K}_t$ we get $\frac{D\alpha_{\overline{K}_t}}{Dt} =$ $= \dot{\alpha}_{\overline{K}_t} - \omega\alpha_{\overline{K}_t} + \alpha_{\overline{K}_t}\omega$.

Relative to the form of the laws of the material written in P.8. we make the following remarks:

1. In order to know the material functions with respect to $\overline{K}_t$ it is necessary to know their form with respect to a set of l.c.r.i.c.

2. Generally, the material functions with respect to $\overline{K}_t$ are dependent on β_t. Therefore the employement of the alternative procedure ofered by P.8. requires the knowledge of the temporal evolution of β_t.

3. This alternative procedure is sugested by the following Mandel's remark: "The relaxed configuration K_t is defined besides a rigid rotation β_t, in such a way that E and P are defined besides an orthogonal transformation. Consequently, P can be replaced by $\beta_t P$ and E by $E\beta_t^T$, if $\beta_t^T\beta_t = I$".

We consider that this conclusion does not take into account the real significance of the elastic deformation which is the reversible deformation of the crystalline lattice, as well as the real significance of the permanent plastic deformation, representing that part of the total deformation which is produced by the motion of dislocations and therefore has a permanent character.

Consequently, the alternative procedure sketched in

P.8. is not usefull in general, it generates confusions and the real physical significance of the elastic and plastic part of total deformations respectively is obscured.

4. From P.8. it is straightforward that in the case of structural isotropic materials (defined by material functions with respect to I_{K_o}, which are isotropic with respect to all their arguments) the material functions do not depend explicitly on β_t. Consequently, it is only in this case that, the employment of Mandel's alternative procedure can be useful.

5. From the above follows that the Lee's alternative, which corresponds to the choice $\beta_t = R^e$ is also useful in the case of structural isotropic materials only.

6. It is necessary to distinguish between the meaning of the objectivity assigned by Mandel to the derivatives introduced in P.8 (i.e. they are independent of the rigid motion of the director frame) and the objectivity relative to a change of the frame in the actual configurations.

The Lee's choice realises both these objectivities for the rate of plastic deformation, as well as for the rate of elastic deformation", if they are "defined" by:

$$\frac{D\bar{P}}{Dt}\,\bar{P}^{-1} = \dot{\bar{P}}\bar{P}^{-1} - \omega, \quad \frac{D\bar{V}^e}{Dt} = \dot{\bar{V}}^e - \omega\bar{V}^e + \bar{V}^e\omega,$$

since, if $Q(t) \in \mathcal{O}$ characterises a change of frame, we have

$$\frac{D\bar{P}^*}{Dt}(\bar{P}^*)^{-1} = Q\,\frac{D\bar{P}}{Dt}\,Q^T, \quad \frac{D\bar{V}^{e^*}}{Dt} = Q\,\frac{D\bar{V}^e}{Dt}\,Q^T.$$

Generally, such kind of relations do not take place for another choice of β_t , since there is no relation between the change of frame with respect to which the motion is refered and the choice of the relaxed configurations with respect to which we describe the behaviour of the material. This fact is completely misunderstood by Casey, Naghdi (1980, 1981) and Sidoroff (1970b, 1971).

7. If the material is structurally isotropic and moreover the elastic stretch are small during the given process, then the alternative procedure from P.8. allows the characterization of the material response via the quantities with respect to the current configuration (since it is possible to substitute π by T in the evolution equations).

8. Let us remark that the hypothesis according to which the elastic deformations remain small while the plastic deformations become very large may not have a real significance since the evolution in time of these deformations is governed by the material laws apriori postulated. Therefore for a given deformation process these material laws can lead to elastic deformations of comparable magnitutde with the plastic ones.

It is not excluded howener that the well known anomalies (see the following section) are due just to the omission of the above mentioned fact and to the replacement of the initial realistic elasto-plastic model with another simplified rigid-plastic one.

9. The real mechanical meaning and usefulness of the various kinds of the multiplicative decomposition of F, intro-

duced by Sidoroff (1973), Dafalias (1983a, 1985a), Levitas (1983) and of the various derivatives associated with these decompositions in order to formulate correctly the material laws are not clear, if we take into account the remarks (1)-(8).

This remark is made since we consider that the employment of the set of the l.c.r. isoclinic configurations is unvoidable when we wish to formulate the constitutive laws in the framework of the model analysed here.

At the same time we claim that the using of another alternative procedures are relevant for certain particular cases only.

We consider that these special cases can be correctly described if and only if the set of l.c.r. isoclinic configurations is taken into account.

3.2. The plastic spin and the anisotropic hardening of a structural isotropic material

As a first answer to the question left open by Mandel (1982), concerning "the quantitative effects of the plastically induced rotation on the response of the plastically deformed structural isotropic solids with anisotropic hardening" was given independently by Loret (1983, 1985) and Dafalias (1983, 1985).

Subsequently we present and discuss according to our point of view their models and principal results.

As internal variables we consider $\alpha^1_{K_t}=\Sigma$ and $\alpha^2_{K_t}=\varkappa$, where Σ is the back stress tensor of Piola-Kirchhoff type with respect to l.c.r.i.c. $K_t \varepsilon I_{K_o}$, and $\varkappa$ is the equivalent plastic strain[1)] with respect to K_t; Σ is a traceless and symmetric tensor.

As Dafalias and Loret have done we suppose that the material is rate independent, the thermal effects are neglected and only isothermal processes are considered. We also assume the temporal invariance condition (which is tacitly assumed by the mentioned authors). According to our axioms we assume the following constitutive relations:

$$\begin{aligned} &\pi=h_{K_o}(C^e,\Sigma,\varkappa)\\ &\dot{P}P^{-1}=\langle\lambda\rangle B_{K_o}(\pi,\Sigma,\varkappa)h(\mathcal{F}_{K_o})\\ &\dot{\Sigma}=\langle\lambda\rangle M_{K_o}(\pi,\Sigma,\varkappa)h(\mathcal{F}_{K_o})\\ &\dot{\varkappa}=\langle\lambda\rangle m_{K_o}(\pi,\Sigma,\varkappa)h(\mathcal{F}_{K_o}) \end{aligned} \tag{17}$$

The yield surface is given by

$S(t)=\{\pi \mid \mathcal{F}_{K_o}(\pi,\Sigma,\varkappa)=0\}$, the plastic factor λ being defined by

$$\lambda=\partial_\pi \mathcal{F}_{K_o}(\pi,\Sigma,\varkappa).\dot{\pi} \text{ on } S(t)$$

with $h(\mathcal{F})=1$ for $\mathcal{F}=0$ and $h(\mathcal{F})=0$ for $\mathcal{F}<0$.

1) $\varkappa$ is not an internal state variable in Kröner's sense.

We recall that I_{K_o} represents the set of l.c.r.i. configurations corresponding to K_o which is arbitrary, but fixed.

We also have $F=EP$, $C^e=E^TE$, $E=V^eR^e$ (where R^e is the elastic rotation and V^e is the pure elastic strain). We pass to the Lee's choice and we introduce the local current relaxed configurations $\bar{K}_t=R^eK_t$ (with $\beta_t=R^e$) and the "plastic" deformation $\bar{P}=R^eP$. Therefore we get $F=V^e\bar{P}$. Using the Piola-Kirchhoff tensors $\bar{\pi},\bar{\Sigma}$, the rate of "plastic" deformation $\frac{D\bar{P}}{Dt}\bar{P}^{-1}$ with respect to the Lee's type configuration $\bar{K}_t$ and the pure elastic strain $\bar{C}^e=(V^e)^2$, we have

$$\bar{\pi}=R^e\pi(R^e)^T,\ \bar{\Sigma}=R^e\Sigma(R^e)^T,\ \bar{C}^e=R^eC^e(R^e)^T, \tag{18}$$

$$\frac{D\bar{P}}{Dt}\bar{P}^{-1}=R^e\dot{P}P^{-1}(R^e)^T, \text{ where } \frac{D\bar{P}}{Dt}=(\dot{\bar{P}}-\omega\bar{P}), \text{ with } \omega=\dot{R}^e(R^e)^T$$

and

$$\frac{D\bar{\Sigma}}{Dt}=\dot{\bar{\Sigma}}-\omega\bar{\Sigma}+\bar{\Sigma}\omega.$$

The laws of the material expressed with material functions relative to $\bar{K}_t$ take the following form (see §2.3):

$$\begin{aligned}
&\bar{\pi}=h_{\bar{K}_t}(\bar{C}^e,\bar{\Sigma},\bar{\varkappa},R^e)\\
&\frac{D\bar{P}}{Dt}\bar{P}^{-1}=\langle\bar{\lambda}\rangle B_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)h(\mathcal{F}_{\bar{K}_t})\\
&\frac{D\bar{\Sigma}}{Dt}=\langle\lambda\rangle M_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)h(\mathcal{F}_{\bar{K}_t})
\end{aligned} \tag{19}$$

$$\dot{\bar{\varkappa}}=\langle\bar{\lambda}\rangle m_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)h(F_{\bar{K}_t})$$

where we have assumed that the two scalar measures of the equivalent plastic strain: $\varkappa$ with respecto to K_t and $\bar{\varkappa}$ with respect to $\bar{K}_t$, are equal, i.e.

$$\bar{\varkappa} = \varkappa \tag{20}$$

From (17)-(20) it follows that the material functions involved in (19) are defined by the relations

$$\begin{aligned}
&h_{\bar{K}_t}(\bar{C}^e,\bar{\Sigma},\bar{\varkappa},R^e)=R^e h_{K_o}((R^e)^T\bar{C}^e(R^e)^T,(R^e)^T\bar{\Sigma}R^e,\bar{\varkappa})(R^e)^T,\\
&M_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)=R^e M_{K_o}((R^e)^T\bar{\pi}R^e,(R^e)^T\bar{\Sigma}R^e,\bar{\varkappa})(R^e)^T,\\
&m_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)=m_{K_o}((R^e)^T\bar{\pi}R^e,(R^e)^T\bar{\Sigma}R^e,\bar{\varkappa}),\\
&F_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e)=F_{K_o}((R^e)^T\bar{\pi}R^e,(R^e)^T\bar{\Sigma}R^e,\bar{\varkappa}),\\
&\bar{\lambda}=\partial_{\bar{\pi}}F_{\bar{K}_t}(\bar{\pi},\bar{\Sigma},\bar{\varkappa},R^e).\frac{D\bar{\pi}}{Dt},\quad \frac{D\bar{\pi}}{Dt}=\dot{\bar{\pi}}-\omega\bar{\pi}+\bar{\pi}\omega
\end{aligned} \tag{21}$$

From (21) it follows that if the constitutive relations (17), considered by Loret (1983) (which are in agreement with our axiomatic system) are accepted, then the constitutive relations considered by Dafalias (1985) , Eqs.(11), (12)) are false.

We consider that our starting point is correct since it takes into account the real significance of elastic deformation (i.e. the reversible deformation of the crystalline lattice) and of the plastic deformation (i.e. the irreversible, permanent deformation produced by the motion of the deffects); (see also Kratochvil (1971, 1972), Mandel (1971, 1972, 1973, 1974), Teodosiu (1970) and Loret (1983)).

Following the procedure initiated by Lee, Liu (1967), (1968) and continuated by Dafalias (1983 a,b,1985a) the definitions of two types of deformations are mixed up. For this reson the Dafalias starting point is generally unaccepted, as being not physically motivated.

If we introduce the additional hypothesis that the material is structurally isotropic (see Mandel (1971c,d,1973, 1974), Loret (1983)) we obtain a considerable simplification of the constitutive relations. Hence (19) and (21) become:

$$
\begin{aligned}
&\bar{\pi}=h_{K_o}(\bar{C}^e,\bar{\Sigma},\bar{\varkappa})\\
&\frac{D\bar{P}}{Dt}\bar{P}^{-1}=\langle\bar{\lambda}\rangle B_{K_o}(\bar{\pi},\bar{\Sigma},\bar{\varkappa})h(\mathcal{F}_{K_o})\\
&\frac{D\bar{\Sigma}}{Dt}=\langle\bar{\lambda}\rangle M_{K_o}(\bar{\pi},\bar{\Sigma},\bar{\varkappa})h(\mathcal{F}_{K_o})\\
&\dot{\bar{\varkappa}}=\langle\bar{\lambda}\rangle m_{K_o}(\bar{\pi},\bar{\Sigma},\bar{\varkappa})h(\mathcal{F}_{K_o})\\
&\bar{\lambda}=\partial_{\bar{\pi}}\mathcal{F}_{K_o}(\bar{\pi},\bar{\Sigma},\bar{\varkappa})\cdot\frac{D\bar{\pi}}{Dt}
\end{aligned}
\qquad (22)
$$

The constitutive functions in (22) are obviously isotropic. The relations (22) are the starting point in the case considered as a general framework by Dafalias (1985a eqs. (11),(12)). After stipulating these equations (which are acceptable only in the structural isotropic case, as we have already mentioned), Dafalias wrote: "Based on the transformation law under superposed rigid body rotation/reflexion, the corresponding invariance requirements render the material functions isotropic functions of their arguments". Similar statements are also given in the papers of Dafalias (1983a, 1985b).

From the above considerations it results clearly that Dafalias assumes from the very beginning the facts he wants and claim to prove.

Dafalias (1983a) writes:"The principale objective of this work is to provide the missing link for the macroscopic application of Mandel's theory, to extend and generalize certain aspects of it and illustrate it by examples.. Our focus will be the macroscopic formulation of constitutive relations for the plastic spin which becomes the key to our initial objective. This is achived by using representation theorems for isotropic functions".

It is obvious that these theorems can be used <u>only</u> if the structural isotropy is <u>assumed</u> from the very beginning, since it can not be provided anyway. Consequently, the Dafalias's reproches to the Mandel's approaches is completely groundless.

The valuable contribution of Dafalias and Loret papers consists, however, in the fact that they show how the material symmetry assumptions combined with the Wang (1970) and Liu (1981) type of representation theorems can theoretically deliver the essential information concerning the structure of the material functions which describe both the plastic rate of strain and the plastic spin.

Before proceeding, we mention that the connection between $\overline{\pi}$, $\overline{\Sigma}$ on the one hand, and of (T,S) on the other hand, where S is a back stress tensor of Cauchy type, is given by the relations $\overline{\pi}=\det V^e V^{e-1} T (V^e)^{-1}$, $\overline{\Sigma}=\det V^e (V^e)^{-1} S (V^e)^{-1}$ as it follows from (3) and (18).

Without any other additional hypotheses the laws of the material can not be simplified. A considerable simplification can be obtained if we suppose that the pure elastic deformation tensor $\varepsilon^e = \frac{1}{2}(V^e - I)$ is small, in the sense that we can neglect in all relations ε^e with respect to the identity and $(\varepsilon^e)^2$ with respect to ε^e. In this case $\overline{\pi}=T, \overline{\Sigma}=S$.

Consequently, all variables involved in (22) can be lather related to the actual configuration, if we assume also that the measure of the equivalent plastic strain $\varkappa$ remains unchanged. With the usual elastic law, we obtain from (22)

$$T=\lambda^e (\mathrm{tr}\,\varepsilon^e) I + 2\mu^e \varepsilon^e, \quad (\lambda^e, \mu^e \text{ - Lame's constants})$$

$$\frac{D\overline{P}}{Dt}\overline{P}^{-1} = \langle\overline{\lambda}\rangle B_{K_o}(T,S,\varkappa)\, h(\mathcal{F}_{K_o})$$

$$\frac{DS}{Dt} = \langle\overline{\lambda}\rangle M_{K_o}(T,S,\varkappa)\, h(\mathcal{F}_{K_o})$$

$$\dot{\varkappa} = \langle\overline{\lambda}\rangle m_{K_o}(T,S,\varkappa)\, h(\mathcal{F}_{K_o})$$

where $\overline{\lambda}=\partial_T \mathcal{F}_{K_o}(T,S,\varkappa)\cdot\frac{DT}{Dt}$ with

$$\frac{DT}{Dt}=\dot{T}-\omega T+T\omega, \quad \frac{DS}{Dt}=\dot{S}-\omega S+S\omega, \quad \omega=\dot{R}^e (R^e)^T.$$

All material functions involved (23) are obviously isotropic functions of their arguments.

We associate to the constitutive system the kinematic relation

$$\dot{F}F^{-1}=L=\frac{D\varepsilon^e}{Dt}+\frac{D\overline{P}}{Dt}\,\overline{P}^{-1}+\omega,\quad \frac{D\varepsilon^e}{Dt}=\dot{\varepsilon}^e-\omega\varepsilon^e+\varepsilon^e\omega \tag{24}$$

which hold when the elastic deformations are small but the temporal rate of elastic deformation is not neglected in the presence of the temporal rate of plastic deformation.

Finally the hardening scalar variable κ and $\overline{\kappa}$ are defined by

$$\kappa(t)=\kappa_o+\int_o^t\sqrt{\tfrac{2}{3}D^p(\tau).D^p(\tau)}\,d\tau,\quad \overline{\kappa}(t)=\overline{\kappa}_o+\int_o^t\sqrt{\tfrac{2}{3}\overline{D}^p(\tau).\overline{D}^p(\tau)}\,d\tau \tag{25}$$

where

$$D^p=\{\dot{P}P^{-1}\}^s,\quad \overline{D}^p=\left\{\frac{D\overline{P}}{Dt}\,\overline{P}^{-1}\right\}^s=R^eD^p(R^e)^T$$

Thus, if we suppose that $\kappa_o=\overline{\kappa}_o$ then $\overline{\kappa}(t)=\kappa(t)$ holds for all t; therefore the hypothesis (20) is satisfied.

The system of equations (23)-(25) can be written in the following equivalent form:

$$\begin{aligned}
&\dot{T}=\omega T-T\omega+\lambda^e\,\mathrm{tr}\,\frac{D\varepsilon^e}{Dt}\,I+2\mu^e\,\frac{D\varepsilon^e}{Dt}\\
&\dot{S}=\omega S-S\omega+\langle\overline{\lambda}\rangle M(T,S,\kappa)h(F)\\
&\dot{\kappa}=\left(\tfrac{2}{3}\overline{D}^p.\overline{D}^p\right)^{1/2}
\end{aligned} \tag{26}$$

$$\frac{D\varepsilon^e}{Dt} = D - \bar{D}^p, \quad \omega = W - \bar{W}^p$$

$$\bar{\lambda} = \partial_T \mathcal{F}(T,S,\varkappa) . \frac{DT}{Dt}, \quad \frac{DT}{Dt} = \dot{T} - \omega T + T\omega$$

$$\bar{D}^p = \langle\bar{\lambda}\rangle \{B(T,S,\varkappa)\}^s h(\mathcal{F}), \quad \bar{W}^p = \langle\bar{\lambda}\rangle \{B(T,S,\varkappa)\}^a h(\mathcal{F}).$$

Using the isotropy of material functions and taking into account the fact that an orthogonal tensor field $\check{R}$ exists such that $\dot{\check{R}}\check{R}^T = W$ ($\check{R}(0) = I$), the relations (26) can be written as a differential system in terms of rotated variables

$$\check{T} = \check{R}^T T \check{R}, \quad \check{S} = \check{R}^T S \check{R}, \quad \check{D} = \check{R}^T D \check{R},$$

$$\dot{\check{T}} = \langle\check{\lambda}\rangle (-\{B(\check{T},\check{S},\varkappa)\}^a \check{T} + \check{T}\{B(\check{T},\check{S},\varkappa)\}^a - \lambda^e \mathrm{tr} B(\check{T},\check{S},\varkappa) I -$$
$$-2\mu^e \{B(\check{T},\check{S},\varkappa)\}^s) h(\check{\mathcal{F}}) + \lambda^e \mathrm{tr}\check{D} I + 2\mu^e \check{D},$$

$$\dot{\check{S}} = \langle\check{\lambda}\rangle (-\{B(\check{T},\check{S},\varkappa)\}^a \check{S} + \check{S}\{B(\check{T},\check{S},\varkappa)\}^a + M(\check{T},\check{S},\varkappa)) h(\check{\mathcal{F}})$$

$$\dot{\varkappa} = \langle\check{\lambda}\rangle (\tfrac{2}{3}\{B(\check{T},\check{S},\varkappa)\}^s . \{B(\check{T},\check{S},\varkappa)\}^s)^{1/2} h(\check{\mathcal{F}}) \qquad (27)$$
$$\equiv \langle\check{\lambda}\rangle m(\check{T},\check{S},\varkappa).$$

with

$$\check{\mathcal{F}}(\check{T},\check{S},\varkappa) = \mathcal{F}(\check{T},\check{S},\varkappa), \quad \check{\lambda} = \partial_{\check{T}} \mathcal{F}(\check{T},\check{S},\varkappa) . \dot{\check{T}}$$

For the integration of the above differential equations (27) we require the values of the plastic factor λ; λ depends on the current value of $\dot{\check{T}}$, which in its turn can be determined only if the value of $\check{\lambda}$ is given at the considered moment.

Hence the system given in the form (27) can not in principle be solved even if the history of F is prescribed. This deadlock can be overcame (see Cleja-Ţigoiu (1983)) by the introducing the complementary plastic factor β, which depends directly on the history of F and consequently λ can be eliminated.

Indeed, in the framework of the accepted approximation from P.4 (§3.3) via the formulae (12) for β and γ we obtain the following relations:

$$\check{\beta}\equiv\beta(t,\check{T},\check{S},\varkappa)=4\lambda^{e}(\mathrm{tr}\ \partial_{\check{T}}\check{F})(\mathrm{tr}\check{D})+2\mu^{e}\partial_{\check{T}}\check{F}.\check{D} \tag{28}$$

$$\check{\gamma}\equiv\gamma(\check{T},\check{S},\varkappa)=2[\partial_{\varkappa}\check{F}.\check{m}+\partial_{\check{S}}\check{F}.\check{M}]-4[\lambda^{e}\mathrm{tr}\{\check{B}\}^{s}.$$

$$.\mathrm{tr}\partial_{\check{T}}\check{F}+2\mu^{e}\partial_{\check{T}}\check{F}.\check{B}].$$

where

$$\check{m}=m(\check{T},\check{S},\varkappa)h(\check{F}),\quad \check{M}=M(\check{T},\check{S},\varkappa)h(\check{F}),$$

$$\check{B}=B(\check{T},\check{S},\varkappa)h(\check{F}).$$

Using also the existing relationship between λ,β and γ (see P.4.b) we can put the system (27) in the form:

$$\dot{\check{T}}=\lambda^{e}(\mathrm{tr}\check{D})I+2\mu^{e}\check{D}+\check{\gamma}^{-1}\langle\check{\beta}\rangle[-\{\check{B}\}^{a}\check{T}+\check{T}\{\check{B}\}^{a}-\lambda^{e}(\mathrm{tr}\check{B})I-$$

$$-2\mu^{e}\{\check{B}\}^{s}]h(\check{F})$$

$$\dot{\check{S}}=\langle\check{\beta}\rangle\check{\gamma}^{-1}[-\{\check{B}\}^{a}\check{S}+\check{S}\{\check{B}\}^{a}+\check{M}]h(F) \tag{29}$$

$$\dot{\varkappa}=\langle\check{\beta}\rangle\check{\gamma}^{-1}[\tfrac{2}{3}\{\check{B}\}^{s}\{\check{B}\}^{s}]^{1/2}h(\check{F})\equiv\langle\check{\beta}\rangle\check{\gamma}^{-1}\check{m}.$$

which principially can be solved if the history of F is prescribed.

In their applications to the general theory, by considering the particular case of a rigid-plastic, rate-independent model, Dafalias and Loret have neglected completely the pure elastic deformations and the temporal rate of the pure elastic deformation with respect to the plastic deformation.

In order to obtain from the general system (26), (27) the particular case of rigid-plastic rate-independent materials (that is the case analysed by Dafalias and Loret) we have to consider that the elastic constants $\lambda^e, \mu^e \to +\infty$. In this case from $(26)_1$ follows $\varepsilon^e = 0$ and (26), (27) are reduced to

$$\begin{aligned}
&D=\overline{D}^p=\langle\overline{\lambda}\rangle\{B(T,S,\varkappa)\}^s h(\mathcal{F})\\
&\omega=W-\overline{W}^p=W-\langle\overline{\lambda}\rangle\{B(T,S,\varkappa)\}^a(\mathcal{F})\\
&\dot{S}=\omega S-S\omega+\langle\overline{\lambda}\rangle M(T,S,\varkappa)h(\mathcal{F})\\
&\dot{\varkappa}=\left(\tfrac{2}{3}D.D\right)^{1/2}\\
&\overline{\lambda}=\partial_T\mathcal{F}(T,S,\varkappa).\ \frac{DT}{Dt}\ ,\ \frac{DT}{Dt}=\dot{T}-\omega T+T\omega
\end{aligned} \qquad (30)$$

Dafalias and Loret consider the following particular form for the material functions

$$\begin{aligned}
&\mathcal{F}(T,S,\varkappa)=\frac{1}{2k^2}(T'-S).(T'-S)-1,\quad T'=T-\frac{1}{3}(\mathrm{tr}\ T)I,\\
&B(T,S,\varkappa)=\frac{\nu}{k^2}(T'-S)+\eta(ST'-T'S)\\
&M(T,S,\varkappa)=c(T'-S)-\delta S
\end{aligned}$$

where k, ν, η, c and δ are scalar isotropic material functions which depend on the fundamental set of scalar invariants of T, S, $\varkappa$. Here the symmetric part of B is just the one used in the classical theory of plasticity while the antisymmetric part was obtained independently by Loret (1983) and Dafalias (1983a, 1985a) using the simplest form of the representation theorems for the isotropic functions.

The consistency conduction (11) becomes

$$1-2c-\frac{4}{\sqrt{3}}\frac{\nu}{k^2}\frac{dk}{d\varkappa}+\frac{\delta}{k^2}(T'-S).S=0 \tag{32}$$

We observe that if c, ν, k and δ are depending on $\varkappa$ only, then the consistency condition is satisfied if and only if δ=0.

If we take into account the term containing δ and if we maintain the hypothesis (see Loret (1983)) that c, ν, k and δ are functions of $\varkappa$ alone, then it is necessary to multiplicate $\bar{\lambda}$ by the factor $1/h_0$, where h_0 is dependent on the invariants of (T, S) and $\varkappa$, and h_0 has to be positive.

Thus we replace $(30)_5$ by the modified plastic factor (see Halphen (1975), Loret (1983), Dafalias (1983a, 1985a)):

$$\bar{\bar{\lambda}}=\frac{1}{h_0}\partial_T F.\frac{DT}{Dt} \tag{33}$$

and therefore the following consistency condition

$$h_0=2c+\frac{4}{\sqrt{3}}\frac{\nu}{k^2}\frac{dk}{d\varkappa}-\frac{\delta}{k^2}(T'-S).S \tag{34}$$

yields.

If we suppose that c, ν, k and $\frac{dk}{d\varkappa}$ are non-negative valued functions and $S(0)=0$ then there exists a certain time interval on which obviously $h_\gamma > 0$. However it is not yet clear if the relation $h_\gamma > 0$ generally holds.

3.3. Kinematic hardening and simple shear

Let us analyse first the following particular cases considered by Loret (1983) and Dafalias (1983a, 1985a).

In this case evolution equations (30), (31) and (33) take the following particular form:

$$
\begin{aligned}
& D=D^p=<\bar{\lambda}> \frac{\nu}{k_o^2}(T'-S)h(F), \quad T'=T-\frac{1}{3}(\mathrm{tr}T)I, \\
& W^p=<\bar{\lambda}>\eta(ST'-T'S)h(F) \qquad (35) \\
& \frac{DS}{Dt}\equiv\dot{S}-\omega S+S\omega=<\bar{\lambda}>c(T'-S)h(F). \\
& \dot{\varkappa}=(\tfrac{2}{3}D.D)^{1/2}, \\
& F(T,S)=\frac{1}{2k_o^2}(T'-S).(T'-S)-1 \\
& \omega=W-W^p \\
& \bar{\lambda}=\frac{1}{k_o^2}(T'-S).\frac{DT'}{Dt} \text{ with } \frac{DT'}{Dt}=\dot{T}'-\omega T'+T'\omega.
\end{aligned}
$$

The consistency condition leads to $c=1/2$. The material function ν can be determined from uniaxial tensile text. In this case we assume that all the stress components but a single one are zero, say $T_{11}=\sigma$.

Thus $T'_{11}=\frac{2}{3}\sigma$, $T'_{22}=T'_{33}=-\frac{1}{2}T'_{11}$, and from the symmetry considerations $S_{22}=S_{33}= -\frac{1}{2}S_{11}$. The yield condition becomes $T'_{11}-S_{11}=\frac{2}{\sqrt{3}}k_o$, if we consider $T'_{11}>S_{11}$.

Since the pure elastic deformations was neglected and $R^e=I$, by considering also the incompressibility condition we obtain that F and P are equal to

$$F = P = \mathrm{diag}\,\{p, 1/_{\sqrt{p}}, 1/_{\sqrt{p}}\}.$$

Therefore

$$L=\dot{F}F^{-1}=\dot{P}P^{-1}=D=D^p=\mathrm{diag}\{\dot{p}/_p, -\dot{p}/_{2p}, -\dot{p}/_{2p}\} \tag{37}$$

Is straightforward that $\bar{W}=W^p=0$, that $(35)_2$ are identically satisfied and that $\omega=0$. Consequently,

$\frac{DS}{Dt}=\dot{S}$ and $\frac{DT'}{Dt}=\dot{T}'$. From $(35)_1$ and (36) we get $\dot{p}/_p=\frac{2}{\sqrt{3}}\frac{\nu}{k_o}<\bar{\lambda}>$ and from $(35)_7$ with $(35)_1$ we obtain $\dot{\varkappa}=2/_{\sqrt{3}}\frac{\nu}{k_o}<\bar{\lambda}>$. Thus $\dot{p}/_p=\dot{\varkappa}$ and

$$\varkappa = \ln p \tag{38}$$

Finally the first evolution equation $(35)_5$ is reduced to $\dot{S}_{11}=\frac{1}{2}<\bar{\lambda}>(T'_{11}-S_{11})$ or to $\dot{S}_{11}=k_o^2\frac{c}{\nu}\dot{\varkappa}$. As we suppose $\dot{\varkappa}>0$ we obtain $\frac{dS_{11}}{d\varkappa}=\frac{k_o^2}{2\nu}$. Combining this equation with the yield condition we obtain that $T_{11}=\sigma$ satisfies the differential equation

$$\frac{d\sigma}{d\varkappa} = \frac{3}{4\nu} k_o^2 .$$

Hence the factor $1/_\nu$ characterizes the hardening of the material. In the particular case of linear hardening when $\sigma=\sqrt{3}k_o+$ $+E_1\ \varkappa$, ν becomes equal to $\frac{3k_o^2}{4E_1}$. Under the above hypotheses all the material coefficients presented in (35) can be determined, but η.

Further, with the same assumed restricted conditions and following Dafalias (1985a) we study the behaviour of the material subjected to a pure shearing due to a continuous loading process.

We suppose that:

1) only one component of F is non zero, i.e. $F_{12}=\gamma(t)$ with $\gamma(t)>0$ for any t;

2) the hydrostatic pressure is zero, i.e. $T'=T$;

3) the initial data T_o and S_o are: $T_{i3}(0)=S_{i3}(0)=0$, $i\epsilon\overline{1,3}$.

It follows from 1) that the non-zero component of L is $L_{12}=\dot{\gamma}$.

Consequently, the non-zero components of D and W are $D_{12}=\dot{\gamma}/2=D_{21}$ and $W_{12}=-W_{21}=\frac{\dot{\gamma}}{2}$.

It follows from $(35)_1$ that $\frac{\dot{\gamma}}{2}=\langle\lambda\rangle \frac{\nu}{k_o^2}(T_{12}-S_{12})$ and for all the other components $T_{ij}=S_{ij}$.

From the hardening condition yields

$T_{12}-S_{12}=k_o$, if $T_{12}-S_{12}\geq 0$

Thus

$$\bar{\lambda} = \frac{k_o}{2\nu}\dot{\gamma}>0$$

Using 2) we get $T_{22}(t)=-T_{11}(t)$, $S_{22}(t)=-S_{11}(t)$. Therefore with the notation $x=S_{11}$, $y=S_{12}$ the evolution equations, which are not identically satisfied, take the following simplified form:

$$\frac{dx}{d\gamma} = y-axy, \quad \frac{dy}{d\gamma} = -x+ax^2+b \tag{39}$$

where $a=\frac{2k_o^2\eta}{\nu}>0$, $b=\frac{k_o^2}{4\nu} \equiv \frac{h_\alpha}{3}>0$, with $x(0)=x_o$, $y(0)=y_o$, if $(35)_{4,8}$ are also used.

In order to study the béhaviour of the material corresponding to the simplified model employed here we have to analyse the solution of the system (39) for $\gamma\geq 0$. Obviously the uniqueness and existence conditions for the system (39) with given initial data (x_o,y_o) are fulfilled.

The critical points of the system are determined from the algebraic system:

$$y-axy=0, \quad -x+ax^2+b=0 \tag{40}$$

The solutions are real if and only if $1-4ab\geq 0$. In this case the solutions are $(x_-=\frac{1}{2a}(1-\sqrt{1-4ab}, 0)$ and $x_+=\frac{1}{2a}(1+\sqrt{1-4ab},0)$ and $0\leq x_-\leq x_+$. It is obvious that the integral curves of the system (39) are symmetric with respect to 0x. On the other hand these curves can be explicitly obtained by integration of the system

$$\frac{dy}{dx} = \frac{-x+ax^2+b}{y(1-ax)} \qquad \text{for } x \neq \frac{1}{a} ,$$

and thus

$$\frac{1}{2}(x^2+y^2)=-\frac{b}{a}\ell n|1-ax|+\text{const.} \qquad (41)$$

A particular solution of the differential equation is the straight line $x=\frac{1}{a}$. The global solutions (41) of the system are defined if the integration constant is determined from the initial condition.

The analysis shows that the critical point $(x_-,0)$ is a center, but $(x_+, 0)$ is a saddle point. The existence of two curves U_1 and U_2 follows. They tend asymptotically towards $(x_+, 0)$ when $\gamma\to\infty$, and $U_1 \cup U_2 \cup \{x_+,0)\}$ is a continuous curve (see Fig.4). On the other hand two symmetric curves (see Fig.4) V_1 and V_2 exists, which tend assymptotically towards $(x_+, 0)$ when $\gamma\to-\infty$. $U_2 \cup V_2 \cup \{(x_+,0)\}$ generate a closed curve, and $(x_-,0)$ with $x_->0$ is an inner point of the domain bounded by this curve. $(0,0)$ is an interior point of the above mentioned domain if and only if

$$(x_+)^2 - \frac{2b}{a}\ln \frac{1}{1-(x_+)b} > 0.$$

The last condition can be satisfied for small η.

Fig.4

From the physical point of view, all the subsequently analysed cases lead to unacceptable solutions.

If the initial value (x_o,y_o) belongs to the interior of the mentioned domain, then the stresses are periodical functions of γ. If (x_o,y_o) lies on the curves U_2 then $T_{12}\to 0$ but T_{11} has a finite limit when $\gamma\to\infty$.

If (x_o,y_o) lies outside of this domain then we obtain again an unaceptable behavour, since there are cases when T_{12} descreeases when γ increases and in all cases T_{11} asymptotically tends to a finite limit.

Finally, for the initial value (x_o,y_o) with $x_o > \frac{1}{a}$, T_{11} is descreasing, too, when γ tends to ∞ and T_{11} asymptotically tends towards a finite value.

In the particular case when $1-4ab=0$ (i.e. $x_-=x_+$) the behaviour of the solution (see Fig.5) is also unacceptable.

If $1-4ab<0$ (see Fig.6) there is no periodical solution, while T_{12} grows unlimited with γ and T_{11} asymptotically tends towards a finite limit which is independent on the initial condition.

After this brief presentation of the principal results due to Dafalias, we can conclude that the model obtained as a consequence of the simplications pointed out above can not be accepted from the mechanical point of view.

Fig.5 Fig.6

Following Dafalias (1985b) (see also Loret (1985)) we subsequently analyse the second example in which the material hardens kinematically but $\delta\neq 0$. In this case, in order to satisfy the compatibility condition (32) it is necessary to introduce a new material function h, depending on the scalar invariants of (T',S). We also assume that the material functions have the following simple form $c=\frac{c_o}{h_o}$, $\nu=\frac{\nu_o}{h_o}$, $\delta=\frac{\delta_o}{h_o}$, $\eta=\frac{\eta_o}{h_o}$, where c_o, ν_o, δ_o are material constants, with h expressed by

$$h_0=2c_o-\frac{\delta_o}{k_o^2}(T'-S).S \tag{42}$$

according to (32).

In this case the system of equations (35) becomes:

$$\begin{aligned} &D=D^p=\langle\bar{\lambda}\rangle\frac{\nu_o}{k_o^2 h_o}(T'-S)h(F)\\ &W^p=\langle\bar{\lambda}\rangle\frac{\eta_o}{h_o}(ST'-T'S)h(F)\\ &\dot{S}-\omega S+S\omega=\langle\bar{\lambda}\rangle\{\frac{c_o}{h_o}(T'-S)-\frac{\delta_o}{h_o}S\}h(F)\\ &F(T,S)=\frac{1}{2k_o^2}(T'-S).(T'-S)-1\\ &\omega=W-W^p,\quad \bar{\lambda}=\frac{1}{k_o^2}(T'-S).(\dot{T}'-\omega T'+T'\omega) \end{aligned} \tag{43}$$

The relations (36), (37) remain unchanged but

$$\bar{\lambda}=\frac{k_o}{2\nu_o}h_0\dot{\gamma}\quad\text{with } h_0=2c_o-\frac{2\delta_o}{k_o}S_{12} \tag{44}$$

Following a similar approach as used above the differential evolution equations can be reduced to

$$\frac{dx}{d\gamma} = \varphi x + (1-\alpha x)y, \qquad \frac{dy}{d\gamma} = \beta - \varphi y + (1-\alpha x)x \tag{45}$$

with

$$\alpha = \frac{2k_o^2}{\nu_o}\eta_o, \quad \varphi = \frac{k_o\delta_o}{2\nu_o}, \quad \beta = \frac{k_o^2 c_o}{2\nu_o}$$

Analysing the system (45) Dafalias (1985b) comes to the conclusion;

a) there are values of the material constants for which the system admits only one equilibrium point;

b) depending on a certain relation between the material constants this point can be a stable nodus or a focus;

c) in the first case there are no oscillations for T_{11} and T_{12}, but in the second case there are stress oscillations.

Consequently this model is a more realistic approach. The stress response presents an assymptotic convergence towards an equilibrium point (a stable spiral in stress space, for some numerical values) and the induced stress oscillations fade away as the equilibrium point is approached with increasing γ.

Dafalias does not analyse the constency condition (32) and it is not clear if the relation $h>0$ holds or not.

Subsequently we deal also with a simple shear problem for kinematic hardening but for elasto-plastic material with small pure elastic deformation and the case when the elastic rate of strain are also taken into account in the presence of the plastic rate of strain (see Cleja-Ţigoiu (1988)).

Therefore the system (26) replaces (35) with the material functions given by (31) with $\delta=0$, $k=k_o$(const.), $c=1/2$ (from the consistency condition (32)). Therefore, if we know the history of homogeneous deformation $t\to F(t)$ the stress state T and the back stress S can be determined, for any fixed X, by solving the following differential equations:

$$\begin{aligned}
&\dot{T}'=\omega T'-T'\omega+2\mu^e D-2\mu^e <\bar{\lambda}> \frac{\nu}{k_o^2}(T'-S)h(F)\\
&\dot{S}=\omega S-S\omega+<\bar{\lambda}>\frac{1}{2}(T'-S)h(F)\\
&\omega=W-<\bar{\lambda}>\eta(ST'-T'S)h(F) \qquad (46)\\
&F(T,S)=\frac{1}{2k_o^2}(T'-S).(T'-S)-1,\\
&T'=T-\frac{1}{3}(trT)I, \quad trT=9KtrD
\end{aligned}$$

in which $\bar{\lambda}$ is replaced by the complementary plastic factor β (see formulae (12)[1)]

$$<\bar{\lambda}>=<\beta>\Gamma^{-1} \quad \text{with } \beta=\frac{2\mu^e}{k_o^2}(T'-S).D, \quad \Gamma=1+\frac{4\mu^e\nu}{k_o^2} \qquad (47)$$

where $D=\{\dot{F}F^{-1}\}^s$, $W=\{\dot{F}F^{-1}\}^a$, and K is the elastic bulk modulus.

We add also the initial conditions $T(0)=T_o$, $S(0)=S_o$.

In the simple shear problem, i.e. for $F(t)=\gamma(t)e_1\times e_2+I$, with $\gamma(0)=0$, the orthogonal transformation $\check{R}$ introduced by (27) corresponds to a rotation of angle $\theta=\gamma/2$ around e_3 and we get for the rotated variables $\check{T}=\check{R}^T T'\check{R}$, $\check{S}=\check{R}^T S\check{R}$ the fol-

1) We replace γ by Γ in order to keep γ for shear strain.

lowing equivalent system:

$$\dot{\check{T}}=2\mu^e\check{D}-\frac{2<\beta>}{\Gamma}\eta\{(\check{S}\check{T}-\check{T}\check{S})\check{T}\}^s h(\mathcal{F})-\frac{<\beta>}{2\Gamma}(\Gamma-1)(\check{T}-\check{S})h(\mathcal{F}) \tag{48}$$

$$\dot{\check{S}}=-2\frac{<\beta>}{\Gamma}\eta\{(\check{S}\check{T}-\check{T}\check{S})\check{S}\}^s h(\mathcal{F})+\frac{<\beta>}{2\Gamma}(\check{T}-\check{S})h(\mathcal{F})$$

with $\beta=\frac{2\mu^e}{k_o^2}(\check{T}-\check{S}).\check{D}$, $\Gamma=1+\frac{4\mu^e\nu}{k_o^2}$, for $\check{D}=\check{R}^T D\check{R}$.

The simple shear problem requires the determination of $\check{T}$ and of the back stress tensor $\check{S}$ which must satisfy the system (48) when the simple shear deformation process and some initial values of $\check{T}$ and $\check{S}$ are given (for instance $T(0)=0$, $S(0)=0$).

With the models in which the plastic factor β is also involved is possible to take into account any homogeneous deformation process, hence any simple shear process, unlike the rigid-plastic models considered by Dafalias and Loret in which shear rate must be assumed to be constant.

It follows that the simple shear problem has an unique local solution.

Some numerical examples are analysed in order to illustrate the effect of the plastically induced rotation (described by the dependence on the material parameter η) on the material response subjected to simple shear (see Fig.7 and 8).

Fig.7 Fig.8

The differences between the curves corresponding to different values of Γ, become significant at large strains only. In the case $\eta=0$ we obtain the analytical solution which reveals also the exact nature of the oscillating stress response for large simple shear even when G (the elastic modulus $=\mu^e$) is also taken into account. The Dafalias's solution (1985a) is obtained for $G\to\infty$. It has been shown that in the case of the elasto-plastic simple shear the oscillations may be induced or not also depending on the value of η. Taking into account the plastic rotations the oscillations have been supresed even for kinematic hardening rule.

It is important to point out that the solutions obtained by solving the elasto-plastic simple shear problem are consistent with the physical fundations of the model or with the limits of its applicability (see §1.1), only, if the range of variation of the shear strain γ leads to deformations which remain moderatly large (30%).

We also mention that the solutions are obtained under the hypothesis of small elastic deformations, consequently $T_{12}/2G$ must be small. Since, in the elastic -plastic kinematic hardening model, for sufficiently large values of the material parameter η the elastic deformations may become unrestricted large, in order to have small elastic deformations we must restrict the range of variation of η, as well as the range of variation of γ.

Chap.4. Materials with elastic range

The models of the materials with elastic range were formulated by Pipkin, Rivlin (1965), Owen (1968, 1970, 1974), Šilhavý (1977), Lucchesi, Podio-Guidugli (1986).

Subsequently, we shall present shortly the approach by Lucchesi, Podio-Guidugli (1986)to the rate-independent materials with elastic range.

4.1. Rate independent materials with elastic range

Here we use the same specific concepts of the theory of materials with elastic range and the same symbols as in the paper by Lucchesi, Podio-Guidugli. Their theory relies on concepts which are sometimes only slight modifications of those already introduced in the above mentioned papers concerning the material with elastic range.

A history is a continuous mapping $\hat{F}:[0,1]\to L^+$ and is interpreted as a one parameter family of deformation gradients at a material point fixed once and for all, with respect to a fixed reference configuration. $\hat{F}(0)$ and $\hat{F}(1)$ are its initial and final value, respectively. A history is constant if it has a constant value. We denote by a^+ the history with the constant value a and the history Q is rigid if it takes its values in O. In particular I^+ is a rigid history with value I interpreted as permanent occupancy of the reference configuration and will be called the reference history.

Let H denotes the collection of all histories and

$\mathcal{D}\subset\mathcal{H}$ the set of all histories such that $\hat{F}(0)\in\mathcal{O}$.

For $\hat{F}\in\mathcal{H}$ and for $s\in[0,1]$ the <u>s-section of $\hat{F}$</u> is the history $\hat{F}_s$ such that $\hat{F}_s(s')=\hat{F}(ss')$ for all $s'\in[0,1]$.

A history $\hat{G}$ is said to be a <u>continuation</u> of $\hat{F}$ if there exists a s-section $\hat{G}_s$ such that $\hat{G}_s=\hat{F}$.

Given $A\in L^+$, a <u>continuation up to A</u> of $\hat{F}$ is a continuation $\hat{G}$ of $\hat{F}$ with final value equal to A.

A subset $E\subset L^+$ will be called an <u>admissible set</u> for $\hat{F}$ if

i) E is the closure of an open set; ii) E is arcwise connected; iii) $\hat{F}(1)\in E$, iv) if $\hat{F}(1)\in\partial E$ then there exist two numbers s_1,s_2, $0\leq s_1<s_2\leq 1$, such that $\hat{F}(s)\in\mathring{E}$[1)] for $s\in(s_1,s_2)$ and $\hat{F}(s)\in\partial E$ for $s\in[s_2,1]$.

In particular the final value of a history may lie either in the interior part or on the boundary of E; in the later case the boundary has to be reached from the interior.

For E an admissible set of $\hat{F}$, the collection of all continuation of F which remains in E will be denoted by $\mathcal{C}(\hat{F},E)$.

Lucchesi, Podio-Guidugli's approach is developed within the framework of the history type theories. The material response to a given history of deformation is described by the constitutive functional:

$$\hat{T}:\mathcal{D}\to\mathcal{S},\quad T=\hat{T}(\hat{F}), \tag{49}$$

1) $\mathring{E}$ denotes the interior of E.

where T is the Cauchy stress at the end of the history $\hat{F}$.

The choice of the constitutive functional $\tilde{T}$ is restricted by the requirements which will be listed below.

A set E is an elastic set for $\hat{F}$ corresponding to $\tilde{T}$ if i) E is an admissible set for F, ii) $\tilde{T}$ is path independent on $C(\hat{F},E)$ i.e., $\tilde{T}(\hat{G})=\tilde{T}(\hat{H})$ whenever both $\hat{G}$ and $\hat{H}$ belong to $C(\hat{F},E)$ and $\hat{G}(1)=\hat{H}(1)$.

$\bar{A}$.1. (Existence of the maximal elastic set). For any $\hat{F}$ fixed in $\mathcal{D}$ there exists an elastic set $E(\hat{F})$ such that if E is another elastic set for $\hat{F}$, then $E\subset E(\hat{F})$. $E(\hat{F})$ is called the elastic range of F corresponding to $\tilde{T}$, likewise an element of $C(\hat{F}, E(\hat{F}))$ is called an elastic continuation of $\hat{F}$.

$\bar{A}$.2. (Invariance of the elastic range under the elastic continuation). For $\hat{F}$ fixed in $\mathcal{D}$, $E(\hat{F})=E(\hat{G})$ for $\hat{G}\in C(\hat{F},E(\hat{F}))$.

Let $\hat{F}\in\mathcal{D}$ and $A\in L^+$ be given. For $\hat{G}$ any elastic continuation of $\hat{F}$ up to A we set

$$T^*(A;\ \hat{F})=\tilde{T}(\hat{G}) \qquad (50)$$

As a direct consequence of path independence of $\tilde{T}$ on $C(\hat{F},E(\hat{F}))$, (50) makes sense and defines a mapping $T^*(.,\hat{F}):E(\hat{F})\to$ $\to S$ called the elastic response mapping corresponding to $\tilde{T}$.

In particular for $A=\hat{F}(1)$ we have $T^*(\hat{F}(1),\hat{F})=\tilde{T}(\hat{F})$.

By using $\bar{A}$.2 we get

$$T^*(.,\hat{F}) = T^*(.,\hat{G}) \qquad (51)$$

for any elastic continuation $\hat{G}$ of $\hat{F}$.

As usually, we say that $\tilde{T}$ is frame-indifferent if $\tilde{T}(\hat{Q}\hat{F})=\hat{Q}(1)\tilde{T}(\hat{F})\hat{Q}(1)^T$ for all $\hat{F}\varepsilon\mathcal{D}$ and for all rigid histories $\hat{Q}$.

The following propositions hold (see Lucchesi, Podio-Guidugli (1986)):

P.1. The constitutive functional $\tilde{T}$ is frame-indifferent iff for all $\hat{F}\varepsilon\mathcal{D}$, $\mathcal{E}(\hat{F})$ and $T^*(.,\hat{F})$ have the following properties:

i) $\mathcal{E}(\hat{F})=\{Q^+\mathcal{E}(\hat{F})|Q^+\varepsilon\mathcal{O}\}$

$T^*(QA,\hat{F})=QT^*(A,\hat{F})Q^T$ for all $A\varepsilon\mathcal{E}(\hat{F})$ and $Q\varepsilon\mathcal{O}$

ii) $\mathcal{E}(\hat{Q}\hat{F})=\mathcal{E}(\hat{F})$ for all rigid histories $\hat{Q}$ and

$$T^*(.,\hat{Q}\hat{F})=T^*(.,\hat{F})$$

In what follows, the following axiom will be assumed:

A.3. The constitutive functional $\tilde{T}$ is frame-indifferent.

The set $\mathcal{E}(I^+)$ will be called the initial elastic-range.

The meaning of this set will be pointed out by the proposition:

P.2. For all $Q\varepsilon\mathcal{O}$ i) $\mathcal{E}(Q^+)=\mathcal{E}(I^+)$, ii) $Q\varepsilon\mathring{\mathcal{E}}(Q^+)$.

Therefore the initial value $\hat{F}(0)$ belongs to $\mathring{\mathcal{E}}(I^+)$ for all $\hat{F}\varepsilon\mathcal{D}$.

Also, we must have $\hat{F}(s)\varepsilon\mathcal{E}(I^{t})$ for all $\hat{F}\varepsilon\mathcal{D}$ and for sufficiently small values of s, since $\hat{F}(0)\varepsilon\mathcal{E}(I^{t})$ and $\hat{F}$ is continuous. Moreover, if the image of [0,1] through $\hat{F}$ is not entirely contained in the closed set $\mathcal{E}(I^{t})$, then there exist $s_o\varepsilon(0,1)$ and $\varepsilon>0$ such that

$$\hat{F}(s)\notin\mathcal{E}(I^{t}) \text{ for } s\varepsilon(s_o,s_o+\varepsilon) \qquad (52)$$

We say that during the history $\hat{F}$ the <u>first yielding</u> occurs at the smallest value s_o satisfying (52) for a certain ε.

For a given $\hat{F}\varepsilon\mathcal{D}$ we say that the history $\hat{S}_F$ is an <u>unloading history</u> corresponding to $\hat{F}$ if $\hat{S}_F$ satisfies the following conditions for all $s\varepsilon[0,1]$:

i) $\hat{S}_F(s)\varepsilon\mathcal{E}(\hat{F}_s)$, ii) $T^{*}(\hat{S}_F(s),\hat{F}_s)=0$.

Let $\mathcal{S}(\hat{F})$ be the collection of all unloading histories corresponding to F. It can be shown that:

<u>$\overline{P}$.3.</u> If it is not empty, the set $\mathcal{S}(F)$ must contain at least one history with positive-definite values.

<u>$\overline{A}$.4.</u> (Structure of the collections of unloading histories). For each $\hat{F}\varepsilon\mathcal{D}$, the set $\mathcal{S}(\hat{F})$ contains exactly one positive-definite history. Let $\hat{S}_F^{+}$ be this positive-definite unloading history.

As a consequence of $\overline{A}$.4. and $\overline{P}$.3. we obtain that S_F^{+} is unaffected by the elastic continuation since the following proposition holds:

P.4. Let $\hat{F}\varepsilon\mathcal{D}$ and let $\hat{G}$ be an elastic continuation of $\hat{F}$. Then the constant continuation of $\hat{S}_F^+$ is the positive-definite unloading history corresponding to $\hat{G}$.

It follows that $\mathcal{S}(\hat{F})$ is invariant under a composition with a rigid history, as stated in:

P.5. For all $\hat{F}\varepsilon\mathcal{D}$ and for all rigid histories $\hat{Q}$, the relations $\hat{S}_{QF}^+=\hat{S}_F^+$ or equivalently $\mathcal{S}(\hat{Q}\hat{F})=\mathcal{S}(\hat{F})$ hold.

For any $\hat{F}\varepsilon\mathcal{D}$ the set

$$\mathcal{E}_R(\hat{F})=\{A\varepsilon\mathcal{L}^+ \mid A\hat{S}_F^+\varepsilon\mathcal{E}(\hat{F})\},$$

called the reduced elastic range and the mapping

$$T^o(.,\hat{F}):\mathcal{E}_R(\hat{F})\rightarrow\mathcal{S}, \quad T^o(A;\hat{F})=T^*(A\hat{S}_F^+(1),\hat{F}),$$

called the structural mapping associated with the history $\hat{F}$ can be defined.

$T^o(.,\hat{F})$ may be interpreted "as the mapping delivering the stress in purely "elastic" deformation processes starting from the stress-free configuration reached after unloading along $\hat{S}_F^+$, taken as the reference configuration" (Lucchesi, Podio-Guidugli (1986)), since the stress $T^*(A,\hat{F})$ attained in any elastic continuation up to A of $\hat{F}$ is equal to $T^o(A\hat{S}_F^+(1)^{-1};\hat{F})$.

On the other hand, for any $\hat{F}\epsilon\mathcal{D}$ and for all $s\epsilon[0,1]$

$$\tilde{T}(\hat{F}_s)=T^*(\hat{F}(s);\hat{F}_s)=T^o(\hat{F}(s)\hat{S}_F^t(s)^{-1};\hat{F}_s)$$

$\overline{P}.6.$ The reduced elastic range and the structural mapping associated with $\hat{F}$ have the following properties:

i) $Q^t\subset E_R(\hat{F})=Q^t E_R(\hat{F})$; ii) if $Q\epsilon\mathcal{O}$, $T^o(Q,\hat{F})=0$;

iii) for any $A\epsilon E_R(\hat{F})$ and for any $Q\epsilon\mathcal{O}$, $T^o(QA,\hat{F})=QT^o(A,\hat{F})Q^T$ holds identically.

The material model defined by the above axioms have a solid-like behaviour since the following property holds.

$\overline{P}.7.$ If $A\epsilon L_i$ is such that $T^o(BA,I^t)=T^o(B,I^t)$ for all $B\epsilon E_R(I^t)$ such that $BA\epsilon E_R(I^t)$, then $A\epsilon\mathcal{O}$.

Let us define now, the global structural mapping.

Let N be the set $\cup E_R(\hat{F})$ for all $\hat{F}\epsilon\mathcal{D}$.

For $N\epsilon N$ fixed, the structural mappings $T^o(.,\hat{F})$, associated with the histories $\hat{F}$ such that $N\epsilon E_R(\hat{F})$, are considered. If all therse mappings yield the same stres at N, we denote their cammon value by $T^{\Pi}(N)$. Therefore

$$T^{\Pi}:N\rightarrow S,\quad T^{\Pi}(N)=T^o(N,\hat{F}) \text{ for any } \hat{F}\epsilon\mathcal{D} \text{ such that } N\epsilon E_R(\hat{F}).$$

$\overline{A}.5.$ (Existence of the structural mapping). The structural mapping associated with all histories $\hat{F}\epsilon\mathcal{D}$ define one global structural mapping.

As a direct consequence of $\bar{A}.5$ we obtain that

$\tilde{T}(\hat{F}_s)=T^{\Pi}(\hat{F}(s)\hat{S}_F^+(s)^{-1})$ for all histories $\hat{F}\varepsilon\mathcal{D}$ and for all $s\varepsilon[0,1]$. Consequently, the positive-definite unloading history plays the role of a "permanent deformation history" in the theory of Owen (1968, 1974) and Šilhavý (1977).

$\underline{\bar{P}.8.}$ The global structural mapping has the following properties:

i) $\mathcal{O}\subset\mathcal{N}=\mathcal{O}\mathcal{N}$ ii) if $Q\varepsilon\mathcal{O}$ then $T^{\Pi}(Q)=0$, iii) for any $N\varepsilon\mathcal{N}$ and any $Q\varepsilon\mathcal{O}$, $T^{\Pi}(QN)=QT^{\Pi}(N)Q^T$ holds identically.

$Q\varepsilon\mathcal{O}$ defines a symmetry transformation for the material described by the constitutive functional $\tilde{T}$ if

$\tilde{T}(\hat{F}Q^+)=\tilde{T}(\hat{F})$ takes place for all $\hat{F}\varepsilon\mathcal{D}$.

$\underline{\bar{P}.9.}$ Q is a symmetry transformation iff, for all histories $\hat{F}\varepsilon\mathcal{D}$ and for all $A\varepsilon\mathcal{E}(\hat{F})$ the following relations hold:

i) $\mathcal{E}(\hat{F}Q^+)=\mathcal{E}(\hat{F})Q$, ii) $T^*(A,\hat{F})=T^*(AQ,\hat{F}Q^+)$.

$\underline{\bar{P}.10.}$ Let Q be a symmetry transformation. Then:

i) $\hat{S}_{FQ}^+=(Q^+)^T\ \hat{S}_F^+Q^+$ and $\mathcal{S}(\hat{F}Q^+)=\mathcal{S}(\hat{F})Q$ hold for all $\hat{F}\varepsilon\mathcal{D}$,

ii) $\mathcal{E}_R(\hat{F}Q^+)=\mathcal{E}_R(\hat{F})Q^+$ and $T^o(AQ,\hat{F}Q^+)=T^o(A,\hat{F})$ takes place for all $\hat{F}\varepsilon\mathcal{D}$ and $A\varepsilon\mathcal{E}_R(\hat{F})$.

Finally, the material symmetry transformations are not detectable by T^{Π}, in the sense given by the following proposition:

P.11. If Q is a symmetry transformation then $NQ=\mathcal{N}$, for all $N\varepsilon\mathcal{N}$ $T^{\Pi}(NQ)=T^{\Pi}(N)$.

A material isotropic when the collection of its symmetry transformations is equal to $\mathcal{O}$.

P.12. For isotropic materials

i) the values of $\hat{S}_I^+$ are scalar multiples of I;

ii) for all $\hat{F}\varepsilon\mathcal{D}$, $\tilde{T}(\hat{F})=T^{\Pi}(\hat{F}(1)(\hat{S}_F(1))^{-1})$ yields for all $\hat{S}_F\varepsilon\mathcal{S}(\hat{F})$;

iii) $\hat{\tilde{T}}(\hat{F})=T^{\Pi}(\hat{V}_F(1))$ is satisfied for $\hat{V}_F(1)\varepsilon\mathcal{S}^+$ which is the left product decomposition of $\hat{F}(1)$ under the form $\hat{F}(1)=$ $=\hat{V}_F(1)\hat{R}_F(1)\hat{S}_F^+(1)$.

Finally we present the next two propositions (given by Luchhesi, Podio-Guidugli (1986)) which are listing sufficient conditions on $E(\hat{F})$, $\hat{S}_F^+$ and T^{Π} in order to insure that $\hat{\tilde{T}}$ is frame-indiferent and $Q\varepsilon\mathcal{O}$ is a symmetry transformation for $\hat{\tilde{T}}$.

P.13. For all $\hat{F}\varepsilon\mathcal{D}$ and for all rigid history $\hat{Q}$, we suppose that

i) $E(\hat{F})=\mathcal{O}E(\hat{F})$, $E(\hat{Q}\hat{F})=E(\hat{F})$

ii) $\hat{S}_{QF}^+=\hat{S}_F^+$,

and moreover, for all $Q\varepsilon\mathcal{O}$, $N\varepsilon\mathcal{N}$, we suppose that

iii) $T^{\Pi}(QN)=QT\ (N)Q^T$;

then, the constitutive functional $\hat{\tilde{T}}$ is frame-indifferent.

<u>P.14.</u> Let $Q \in \mathcal{O}$. We assume that

i) $\mathcal{E}(\hat{F}Q^{t}) = \mathcal{E}(\hat{F})Q$

ii) $\hat{S}^{+}_{FQ} = (Q^{T})^{t}\hat{S}^{+}_{F}Q^{t}$

take place for all $F \in \mathcal{D}$ and

iii) $T^{\Pi}(NQ) = T^{\Pi}(N)$

holds for all $N \in \mathcal{N}$.

Then, Q is a material symmetry transformation.

4.2. Connection between the materials with elastic range and the materials having local relaxed configurations

Concerning the theory they have presented Lucchesi and Podio-Guidugli (1986) observe that: "whithin the framework of Axioms $\bar{A}.1$-$\bar{A}.5$, assigning the constitutive functional $\tilde{T}$ is equivalent to assigning, as is done in applications, the elastic range $\mathcal{E}(\hat{F})$ and the positive-definite unloading history $\hat{S}^{+}_{F}$ corresponding to each history $\hat{F} \in \mathcal{D}$, together with the structural mapping T^{Π}".

Starting from this remark, we searched (Cleja-Ţigoiu, Soós (1988)) the connection of our axiomatic model with the model of the materials with elastic range. This comparison is entirely motivated by the fact that (P, α) can be described as solutions of some differential equations which depend on the history of the deformation gradient $\hat{F}$ and on the temperature θ.

Consequently, the elastic deformation $E = FP^{-1}$, as well as α, introduces the dependence on the history of (F,θ) in the material response, via admissible thermoelastic processes (see formulae (4) in §2.2). Here, we restrict ourselves to isothermic processes and to rate-independent materials, i.e. the evolution equations (10) in §2.2 do not contain the viscous terms, $A=0$, $\ell=0$.

From the axioms A.1-A.9, the definitions and the properties given in chap.3, can be proved the validity of the following consequences:

From P4 (§2.2) we obtain

C.1. The complementary plastic factor β is defined by

$$\beta=P^{-1}\,\partial_{C^e}\tilde{F}(\zeta)P^{-T}.\dot{C}^{+}, \qquad C=F^TF$$

and the hardening modulus γ is given by

$$\gamma=2\partial_{C^e}\tilde{F}(\zeta).\{C^eB(\zeta)\}^s + 1.$$

We recall that[1)]

$$\tilde{F}(\zeta)=F(h(C^e,\alpha),\alpha) \quad \text{with } \zeta=(C^e,\alpha).$$

C.2. (see Cleja-Ţigoiu (1988)). From the material law, with C.1, we obtain the strain formulation of the evolution equations for the function $Y=(P^{-1},\alpha)$:

1) We denote by tilda ($\sim$) superposed on the symbol of the considered function, the function obtained by the procedure indicated here.

$$\dot{Y}=\langle\beta\rangle\mathcal{B}(C(t),Y),\quad Y(0)=Y_o \text{ with} \tag{53}$$

$\mathcal{B}(C,Y)=\{-\gamma^{-1}\overline{B}(C,Y),\ \gamma^{-1}\overline{m}(C,Y)\}$ defined on the yield surface $\overline{\mathcal{F}}(C,Y)=0$, with $\partial_Y\overline{\mathcal{F}}(C,Y).\mathcal{B}(C,Y)=-1$. Here by a dush (–) superposed on the symbol of the considered function we denote the function obtained by the procedure indicated below:

$$\overline{B}(C,Y)=\tilde{B}(P^{-T}CP^{-1},\alpha)$$

<u>C.3</u>. If $\hat{F},\mathcal{B},\overline{\mathcal{F}}$ are such that the right hand side of the system (53) satisfies the Peano-Lipschitz conditions on the set $\{(t,Y)\,|\,\overline{\mathcal{F}}(C(t),Y)=0,\ \beta(t,Y)\geq 0\}$ then the Cauchy problem allows an unique local solution.

Let $\mathcal{D}$ be the set of all histories of deformations for which C.3. holds.

From C.2., C.3, for all $F\varepsilon\mathcal{D}$ it follows

<u>C.4</u>. The set $E(\hat{F})=\{F\varepsilon L^+\,|\,\overline{\mathcal{F}}(C,Y(t))\leq 0,\ C=F^TF\}$ is an <u>admissible</u> set for $\hat{F}$.

<u>C.5.</u> $E(\hat{F})=\{C\varepsilon S^+\,|\,\overline{\mathcal{F}}(C,Y(t))\leq 0\}$ and moreover $QE(\hat{F})=E(\hat{Q}\hat{F})=E(\hat{F})$ for $Q\varepsilon\mathcal{O}$, $\hat{Q}$ an orthogonal history.

From C.5 and C.3 it results:

<u>C.6.</u> 1) There exists a stress constitutive functional $\tilde{T}:\mathcal{D}\to S$ which gives the current value of Cauchy stress T corresponding to the history $\hat{F}$ and it is defined by

$$\tilde{T}(\hat{F})=\det P\ \det F^{-1}FP^{-1}h(P^{-T}CP^{-1},\alpha)P^{-T}F^T$$

where $Y=(P^{-1},\alpha)$ is the current value of the solution for the system (53) corresponding to $\hat{F}$ and $F=\hat{F}(t)$.

2) $\hat{T}$ is frame independent, 3) $\hat{T}$ is path independent on $C(\hat{F},E(\hat{F}))$.

From C.4-C.6. it follows:

C.7. The set $E(\hat{F})$ introduced in C.4. is a maximal elastic set, invariant under elastic continuation.

C.8. There exists a global structural mapping

$T^{\amalg}: E\to S$, where $E=\cup E_R(\hat{F})$, $\hat{F}\varepsilon D$, with

$E_R(\hat{F})=\{E\varepsilon L_i \mid EP(t)\varepsilon E(\hat{F})\}$, and defined by

$T^{\amalg}(E)=f_{K_o}(E,\alpha)$ having the properties: 1) $Q\varepsilon E=QE$ for any $Q\varepsilon O$, 2) $T^{\amalg}(A)=0$ iff $A\varepsilon O$, $T^{\amalg}(QE)=QT^{\amalg}(E)Q^T$.

Concerning the material symmetry we state:

C.9. For all $Q\varepsilon g_{K_o}$ 1) the elastic range E and the global structural mapping $T^{\amalg}$ have the properties : $EQ=E$, $T^{\amalg}(EQ)=T^{\amalg}(E)$,

2) the equality $\hat{T}(\hat{F}Q)=\hat{T}(\hat{F})$ holds for all $\hat{F}\varepsilon D$.

As a consequence, in the sense of our definition D.2. (see §2.3), any symmetry transformation is a symmetry transformation in the sense given in §4.1.

We also mention that from: A.4, the continuity of $E\to h(E,\alpha)$ in $E=I$, C.3, and C.4-C.6 follows:

C.10. The set of unloading histories corresponding to $\hat{F}\varepsilon\mathcal{D}$ is a non-empty set, since the history of plastic deformation $\hat{P}$, corresponding to $\hat{F}\varepsilon\mathcal{D}$, is an unloading history (in the sense given in §4.1).

Moreover: for any rigid deformation Q^{+},$Q^{+}\hat{P}$ is itself an unloading history. Consequently for $\hat{F}\varepsilon\mathcal{D}$ there exists exactly one positive-definite unloading history $\hat{V}^{p}$-the pure plastic strain history corresponding to $\hat{P}$.

We observe that $Q^{+}\hat{P}$ is a"permanent history "accordint to Owen (1968, 1974) and an "unelastic history" accordint to Šilhavý (1977),but it is not a plastic deformation history in the sense of our definition (see §2.2).

On the basis of C.10, but taking into account the observation which was just made, we can say (see Owen (1974)) that in an elasto-plastic material the larger the symmetry group the larger the posibilities for "plastic" deformation histories, and that the elastic rotation is arbitrary for a given history F.

Concluding remarks: From the above propositions follows that our model represents a realisation of the theory of materials with elastic range and that the axioms assumed in §2.2 and §2.3 allow us to determine the stress constitutive functional.

In this sense the results presented for the rate-independent materials justify the Kratochvil's conjecture (Kratochvil (1972a)): "The finite strain theory of elasto-plastic materials suggested by Lee, Liu, may be regarded as a special case of Owen's theory".

On the other hand, we consider that the behaviour of elasto-viscoplastic materials can not be modeled within the framework of materials with elastic range, but it can be modeled using c.l.r.i.c. and i.s.v. (see Soós (1983), Cleja-Ţigoiu (1983, 1988), in spite of the statement made by Lee and Germain (1974).

Chap.5. Recent results

5.1. Some theories developed between 1985 and 1988

In this section, we present briefly some results obtained in the interval 1985-1988 and which are related to models based on the local current relaxed configurations. We mention that all considered references were published in the International Journal of Plasticity. We limite ourselves to this journal since we consider that its content reflects well enough the recent work done in the field discussed and analysed in our paper.

Reed and Atluri (1985) remark that the results obtained independently by Lehmann (1972), Dienes (1979), Nagtegal and Jong (1984) concerning the prediction of stress oscillations in simple shear problem, "served to emphasize the need for a more rigorous method of arriving at rate type constitutive equations describing kinematic hardening then in vogue: a mere replacement of the inobjective material stress rate by an objective rate".

Concerning this problem, Reed and Atluri observed: "we do not believe the present search for an "ideal" stress rate for these problems to be well founded. Our dissatisfaction with the model based on the new stress rate stems from the fact they cannot be brought into agreement with any realistic idealization of material behaviour, much less with experimental data. None guarantees stable material behaviour in rectilinear shearing. Normal stresses and normal strains predicted by the models are generally one or two orders of magnitude larger then have been reported in the experimental literature".

By using experimental data and hypoelastic constitutive equations Reed and Atluri proposed a new material law for the back stress S, in which the Jaumann rate of S depends linearly on the plastic rate of strain, but is a non-linear function of S.

The model is a rigid plastic one and a Mises type description of the loading surface and associated flow rule are adopted.

For the normal strain predicted by the model in pure torsion and monotonic loading there is an excellent agreement with Swift's (1947) data. The predictions of the rigid plastic model concerning the shear-stress and the average shear strain and the Swift's data are also in good agreement.

We consider that the model proposed by Reed and Atluri represents one possible choice, but the same agreement can be obtained by using other kind of objective rates, for instance, the one based on plastic spin (Cleja-Ţigoiu (1988)).

In such a way, the problem concerning the description of various kinds of hardening and, of course that of recovery, remains open.

One possible structural approach to this problem is presented by Tokuda, Kratochvil, Ohno (1985). The macroscopic inelastic response is treated as an average of some elementary plastic events. The elements in the slip model considered by the authors are supposed to be single crystal grains in which the inelastic deformations are carried out by crystallographic slips.

The model incorporates interactions among grains in the polycrystal (internal stress) and interactions among the slip systems in a single crystal grain (latent hardening).

The equation for the slip strain in a slip system is based on the theory of thermoactivated motions of dislocations.

The model takes into account the fact that when the polycrystal undergoes a deformation process, a slip starts first in a favourable orriented slip system in a favourably orriented grain. Consequently, the plastic deformation in a polycrystal is generally non-uniform and differs from grain to grain. The single crystal component can not deform freely owing to the interaction of surrounding grains, and the interaction among grains is considered by the authors assuming that the strain state is uniform in the polycrystal, but the plastic and elastic part, respectively, are not uniform.

The numerical computed results are in good agreement with the experimental data in the case of the selected FCC type crystal at elevated temperature and multiaxial strain conditions. In the paper of Tokuda and Yamada (1988) the model was expanded to the large strain range and later we shall comment upon this work.

In contrast with the approach of the above mentioned paper, Anand (1985) deals with a macroscopic approach and developes a set of phenomenological internal variable type constitutive equations describing the elevated temperature deformation of metals. In the paper, a number of typical concepts concerning the formulation of constitutive equations of elastic-plastic materials at finite deformations are tackled: the transformation rules under a change of frame for E and P involved in the decomposition F=EP, the proper choice of frame-indiferent rates, the role of plastic spin, a proper specification of the evolution equations for tensorial internal variables.

All these problems are heatedly debated in the literature and Anand (1985) provides a coherent, physically motivated, mathematically clearly formulated model, which is in accordance with our point of view regarding a theory based on the relaxed configurations.

For the development of the constitutive equations, Anand assumes that with each particle a director triad is associated (assumption steeming from Mandel's) which determines the orrientation of the neighbourhood of the particle. The author employs a conceptual local configuration which is reached by unloading a small material neighbourhood, by reducing the stress to zero and by bringing the temperature back to its initial value. It is important to mention the following Anand's observation: "the unloading process is again conceptual in nature in that we assume: i) that it is possible to fix the current arrangements of material neighborhoods on the microscale so as not to allow any rearrangements by slip diffusion and the like while we reduce the pair (T,θ) for each element to $(0,\theta_o)$; and

ii) that it is possible to orient the unloaded element such that the director triad $\{e_i\}$ in this configuration has the same orientation with respect to an orthogonal basis $\{e_i\}$ in space, as it did in the reference configuration B_o". Following Mandel (and of course Teodosiu and Kratochvil) such an <u>unique</u> current relaxed configuration is called by Anand as "isoclinic". Exactly as in our axiomatic system the deformation associated with this specially oriented relaxed configuration is called the plastic deformation, P. Consequently, Anand defines the elastic deformation by $E=FP^{-1}$. The transformation rules for P and E under a change of frame are the same as those presented by us, in particular P remains <u>unaffected</u> by a change of frame. As an interesting feature of the model, and quite distinct from the usual theories, Anand does not assume the existence of the yield condition as well as of a loading criterion.

These assumptions reflects the experimental feature that at elevated temperatures, the plastic flow may occur at any value of stress and there is no instantaneous plasticity. Also, we observe that Anand tacitly considers a time invariance axiom of the kind used by us, because the dependence or invariance of the material functions on the l.c.r.i.c. is not mentioned anywhere. Consequently, it is not clear the reason for which Anand considers the symmetry group as being "instantaneous", i.e. depending on time.

In accordance with Anand we consider that his paper constitutes a possible answer to the problem posed by Rice (1975): "Given an initially isotropic material in which the plastic state is assumed to be characterized by a scalar field and by a second order tensor field, what is the most general possible class of flow rule and evolution equations."

Also we think that Anand is right when he writes "it is clear that even for the highly idealized class of constitutive equations which use the scalar and symmetric second order tensor as internal hardening variables, much work based on the experiments and considerations of physical mechanism of viscoplastic deformation at elevated temperature needs to be done in order to specify particular forms for material functions which may be suitable for practical applications".

The point of view presented by Haupt (1985) is in complete disagreement with the basic assumptions adopted by Anand. Indeed, starting from ideas developed by Holsapple (1973) within the framework of the models based on history-type constitutive functionals, Haupt considers that the plastic rotation is not related to the history of the mechanical process via a constitutive functional. In other words, he belives that the multiplicative decomposition of the deformation gradient in an elastic and plastic part, respectively, can be assumed on a purely kinematical basis, i.e. without any constitutive assumptions. On this basis, which is wrong from our point of view, Haupt considers that the plastic rotation can be arbitrarly changed by using an additional orthogonal tensor, arbitrarly choosen. Consequently, Haupt assumes that in any process the plastic rotation can be taken as being identically equal with the unit tensor I. As a consequence, he states that: "all constitutive assumptions must be expressed by equations which are frame-indifferent as well as invariant with respect to a superimposed plastic rotation". This statement is in a full agreement with the considerations made by Sidoroff (1971), Casey, Naghdi (1980,1981), but in complete disagreement with our point of view. We consider that the assumptions made by Haupt are completely erroneous because the physical and microscopical basis of plastic deformation shows that, generally the variable plastic rotation exists. Also, from pure phenomenological point of view and in a correctly stated constitutive frame it results that Haupt's assumptions are acceptable only in the particular case of structural isotropic materials, when only scalar type internal variables are taken into account. (See, for instance Dafalias (1983, 1985) and Loret (1983, 1985). Moreover, if we assume that the material considered by Haupt is not plastically deformed, according to his model it must be inevitably elastically isotropic. For us, it is clear that, in reality, this feature is not acceptable.

Similar but unacceptable ideas and results, are presented by Haupt and Tsakmakis (1986) in a subsequent paper devoted to the kinematic hardening at large plastic deformations.

In a paper devoted to "the physical of plastic deformations" Aifantis (1987) deals with the formulation of theories of plasticity at large deformations, based upon an assumption concerning the existence of a set of continuously distributed straight edge dislocations, the carriers of plastic deformation, moving along their slip planes. The author considers that his results provides the microscopic substantiation of various phenomenological proposals for the plastic spin, recently and independently advanced by Dafalias (1983a,b, 1985), Loret (1983) and others.

In order to elaborate a model of elasto-plastic deformation at finite strain, the author begins with certain relationships for the large elasto-plastic deformations. He admits the multiplicative decomposition of the total gradient deformation F, but as the author says, in a slightly different version $F=RU^eF^p$ (44), "where R denotes the rotation of the lattice slip system or the material rotation as opposed to the rotation of the continuum; U^e is the elastic stretch and F^p represents the purely plastic part of the deformation gradient. In usual theories of elasto-plasticity the first two terms of the right hand side of (44) are lumped together and denoting by F^e, the elastic deformation gradient. As this practice may rise some questions concerning the uniqueness of the intermediate relaxed configuration, that is the configuration of the continuum after the removal of F^e we retain the adopted decomposition as our starting point. Assigning R to certain characteristic direction of the material in this case the lattice or slip directions, it removes the above ambiguity and allows for a clear presentation of the main ideas". "Physically R arises from the geometric contraints imposed by the boundary conditions on the slip direction, U^e arises from the usually reversible lattice displacements of elastic nature and F^p arises from permanent or irreversible slipping of crystal portions with respect to each other due to dislocation motion".

Apparently, Aifantis is in good agreement with the point of view addopted by Mandel, Teodosiu, Kratochvil concerning the physical basis of the considered decomposition, though, for us, it is not clear the motivation concerning the meaning of the rotation R. We can not understand the reason for which the term $\dot{R}R^T$ is not included by Aifantis in the elastic spin. Consequently, we consider that the model given by Aifantis is not in agreement with our model as being physically non-motivated.

Also, it is not clear to us the reason for which the kinematic slip system can be treated in a dualistic manner, considering at the same time the components of the slip system as being connected with the crystal lattice and on the other hand as being material elements (see for instance the relations (49) versus (46)).

We also observe that the principal microstructural relations considered by Aifantis for the rate of plastic deformation are in total disagreement with the results given by Tokuda, Yamada (1988).

According to us: a) many of the basic kinematic relations can be accepted only in the cases in which the elastic stretch is equal to I;b) the proof of the final results given by Aifantis (relation (63)) which, according to his opinion, provides a microscopic derivation of a constitutive equation for plastic spin (proposed by Dafalias and Loret) is acceptable within the framework given by the author only when $U^e=I$; c) in order to prove the mentioned result, Aifantis assumes that only one and the same slip system is acctivated in the whole body. However, in this case the material can not be structurally isotropic, although the phenomenological constitutive equations finally obtained by Aifantis have such material symmetry.

The formulation of objective rate-type constitutive relations in finite deformations is considered by Paulum and Pecherski (1987) in a purely phenomenological may.

The authors observe that the theories discussed in most of the papers, related to the problem, lead to an inadequate prediction of material behaviour in the problem of simple shear and single sheas traction. In their paper,the plastic spin concept and the related constitutive equations are discussed for rigid-plastic materials with combined isotropic kinematic hardening. Paulum and Pecherski correctly observe that "it is necessary for adequate description of anisotropic hardening and large plastic deformations to account properly for the material structure and its evolution in the deformation process. The structure description is achieved by introducing the relation between stress, strain and their rates a set of internal structure variables... . These variables represent macroscopically the effects of microstructural rearangements. They are defined, however, directly at the macrolevel and are determined in macroscopic experiments... It was recognized by Mandel (1971, 1972, 1973) and Dafalias (1983b, 1985) that the structure variables are attached to the substructure of the medium and not to the continuum itself. Similarly, the stress is "carried by the structure" of the material. Therefore it seems quite natural and reasonable to define the obejective rates appearing in the constitutive equations by means of rates corotational with the material substructure. Such corotational rates require the definition of the spin tensor which reflects the rate of the rotation of substructure. The thought spin is the difference of the plastic spin from the total spin".

In our opinion, the last sentence, can be accepted only when the elastic deformation is small, the correct formulation of all constitutive and evolution equations requires the use of l.c.r.i.c.

The theory discussed by Paulun and Pecherski is based on a new constitutive relation for plastic spin, and on the use of the spin $\omega=W-W^p$, in order to obtain an objective time rate for stress and for the back stress, as donne by Dafalias and Loret.

The authors compare the theoretical results obtained within the framework of the models elaborated by Lee, Mallet and Weithaimer (1983), Onat (1982), Dafalias (1983a,b, 1985 a,b), Loret (1983, 1985) and Paulun,Pecherski (1985) for the case of the problem of simple shear traction, with the experiments of Swift (1947). On the basis of the results presented by Paulun and Pekerski we consider, in accordance with these authors, that "further studied should be related to the search for the non-linear specification of the constitutive equation for the plastic spin. Solving this problem could shed more light on the general description of anisotropic hardening in finite deformation plasticity".

Also, we consider as being important the following conclusion given by Paulun and Pecherski "although the foregoing discussion follows Read and Alturi (1985) in the applications of the Swift's test for veriffing in the theory, the results depicted in Fig.4 contradict their arguments against "the models based on the new stress-rate", which, according to them, predict normal strains one or two orders of magnitude larger than have been reported in the experimental literature".

Finally, we observe that the model given by Paulun and Pecherski posesses the same principal difficulty commun to almost all of theories concerning elasto-plastic deformations: If the history of stress is given, in order to obtain the evolution of the plastic strain, we must know a priori its evolution.

In a recent paper given by Metzger, Dubey (1987) a new hypoelastic model is associated with an isotropic flow rule to form an elastic-plastic constitutive equation. The use of the principal axes technique ensures that the stress tensor is coaxial with the elastic stretch tensor and that the solution does not depend on the choice of the objective stress rate. Finally, by using the flow rule of von Mises and the parabolic hardening law, a solution is obtained for the prescribed deformation of simples shear. The authors assume the following decomposition $F=VR=V^eV^pR$, where V is the total symmetric stretch, R is the total rotation, V^e is the elastic symmetric stretch and V^p is the plastic, generally not symmetric stretch.

Although Metzger and Dubey write that the derivation of this decomposition was directed by some notion of the physical process of elastic-plastic deformation we consider that a physical basis leading to the decomposition adopted by Metzer and Dubey does not exist. From our point of view the starting point of the authors can not be accepted.

In a recent interesting and important paper, Tokuda and Yamada (1988) derive a set of inelastic constitutive equations for polycrystalline metals by combining a finite deformation kinematics of single crystal components with a shear stress-shear strain relation of slip system based on the thermoactivated motion of dislocations. Interactions among grains are taken into account assuming the deformation gradient in the grain as being constant. By using the thus obtained equations, the effects of grain rotation on the inelastic behaviour of metals are studied theoretically. The results are compared with the available experimental data.

According to us for a phenomenological formulation of a model, by comparing experimental and computational results, the following conclutions due to Tokuda and Yamada (1988) are important: significant differences appear between the stress values when these values are obtained either without taking into account the grain rotation or are obtained by incorporating grain rotation. However, according to our opinion it is very important that such differences mutually cancel out, and the effects of rotation on the macroscopic response are not larger than 50%.

At the same time, Tokuda and Yamada consider the problem of plastic spin discussed by Dafalias (1983 a,b,1985). For Dafalias the missing link between the Mandel's theoretical approach (1971, 1972, 1973) and its practical application is: the procedure to construct concrete constitutive equations for plastic spin Ω^p. The missing link is also due to the existing missing link between macroscopic (phenomenological) theories and microstructural theories of inelastic behaviour of materials. As it is known Dafalias and Loret have obtained phenomenological equations for Ω^p by using the representation theory for isotropic functions. However, as it is shown in the study of Tokuda and Yamada, the spin tensor is different from one grain to another and macroscopic spin can not be obtained by any usual averanging.

We are in perfect agreement with the opinions formulated by Tokuda and Yamada concerning the connection between macroscopic and microscopic theories: "Through the macroscopic spin tensor can be in principle, expressed in a general form by using the representation theory, for example, it may be very difficult to obtain any realistic and concret expression of Ω^p,

Simultaneously, this discussion reveals the difficulty in obtaining a resonable form of tensor rate, i.e. phenomenological inelastic constitutive equations of polycrystaline metals".

Let us observe that there exists a clear contradiction between the realistic results obtained by Tokuda and Yamada and the results given by Aifantis (1987).

We hope that our short review of some papers, dealing with the model presented by us and published in the journal devoted to "plasticity", reveals the unsatisfactory and total confused situation existing in the theory of elasto-plastic deformation today.

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Figure capture

Fig.1: Elastic and plastic deformations

Fig.2: The distinction between elastic and plastic deformations:

(a) reference configuration, (b) pure elastic rotation, (c) pure elastic strain, (d) pure plastic rotation, (e) pure plastic strain

Fig.3: Local current relaxed isoclinic configurations; material symmetry transformations

Fig.4: Integral curves in the simple shear; $1-4ab>0$

Fig.5: Integral curves in the simple shear; $1-4ab=0$

Fig.6: Integral curves in the simple shear; $1-4b<0$

Fig.7: The evolution of the normal stress T_{11} in the simple shear

Fig.8: The evolution of the shear stress T_{12} in the simple shear.

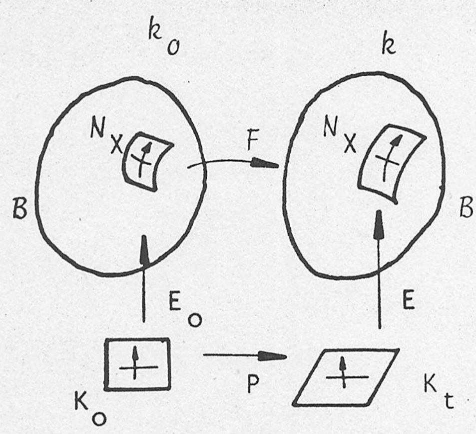


Fig. 1

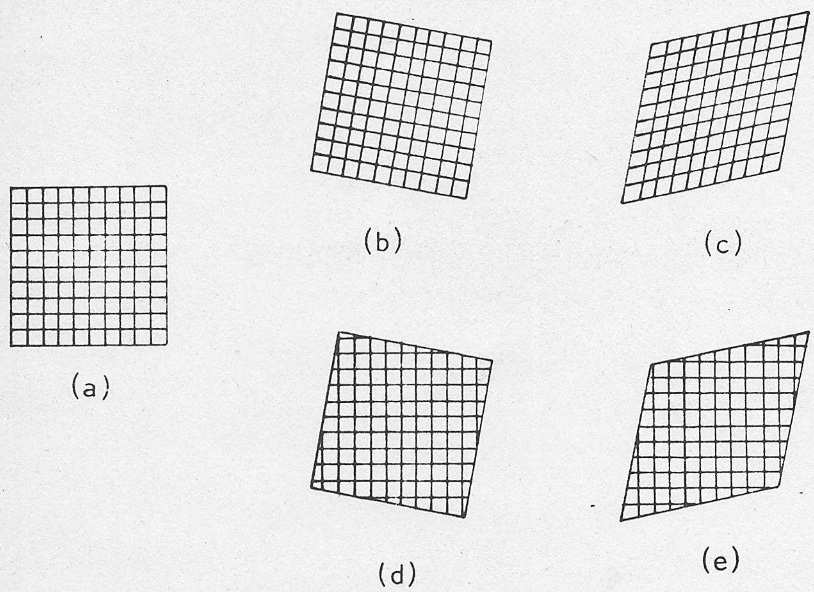
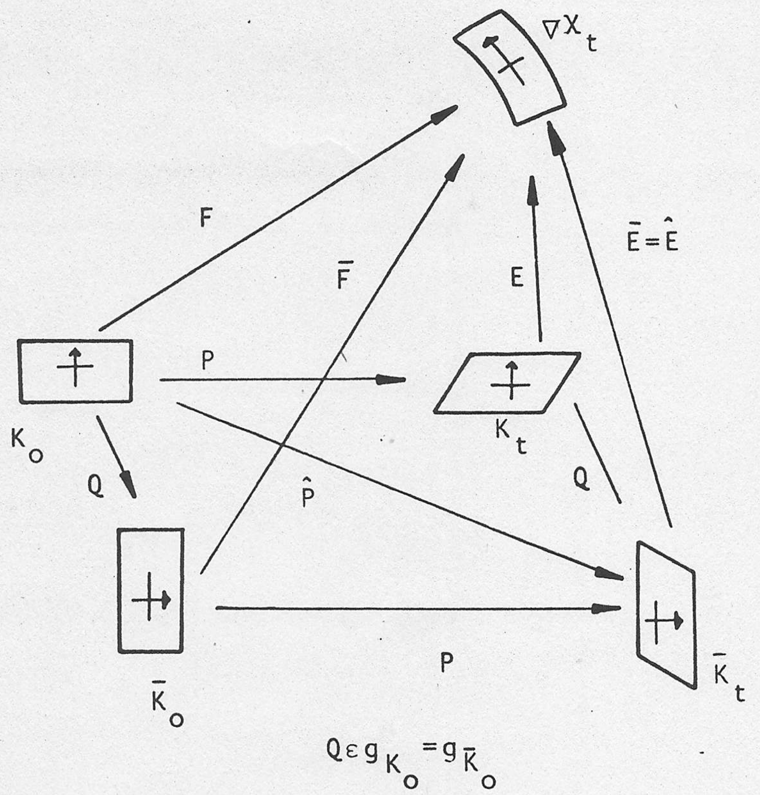
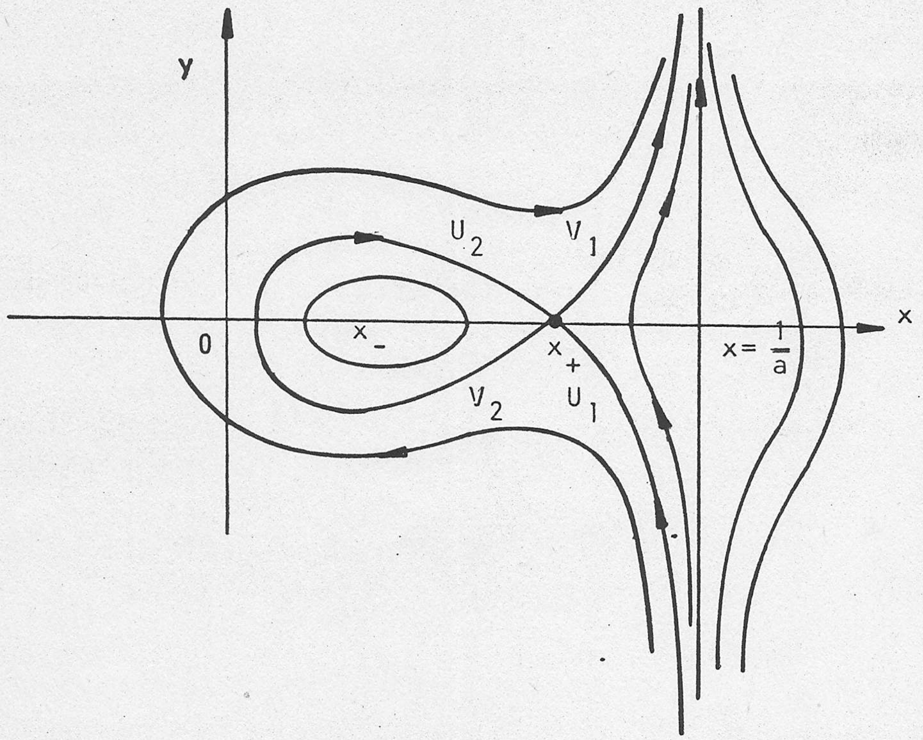


Fig. 2



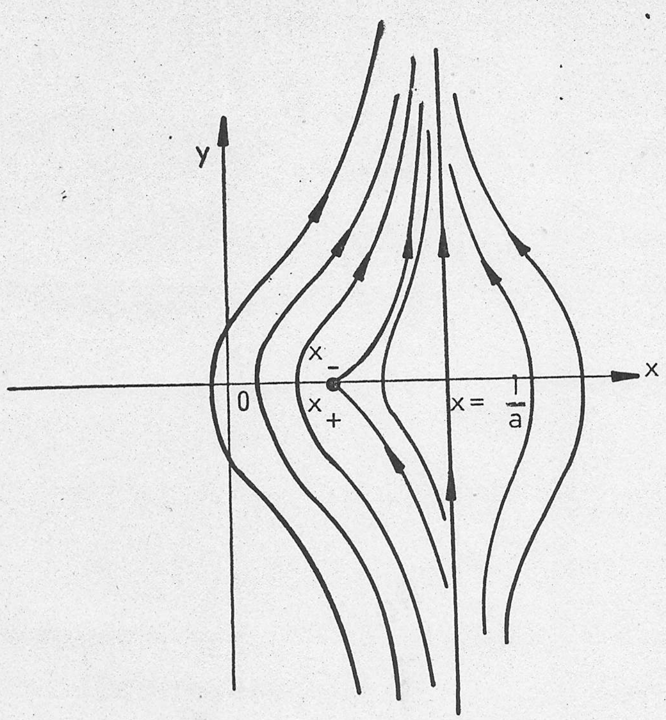
∇X_t
F
$\bar{F}$
E
$\bar{E}=\hat{E}$
P
K_o
K_t
Q
$\hat{P}$
Q
P
$\bar{K}_o$
$\bar{K}_t$
$Q\varepsilon g_{K_o}=g_{\bar{K}_o}$

Fig. 3



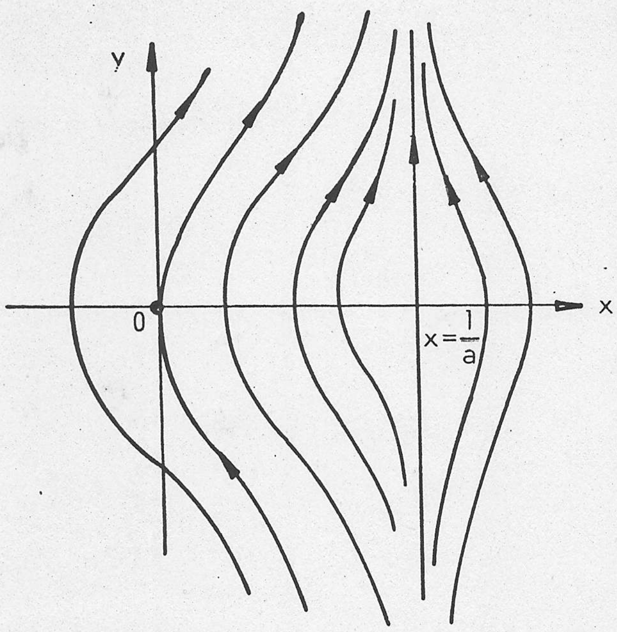
1−4ab> 0

Fig. 4



$1-4ab=0$

Fig. 5



$1-4ab<0$

Fig. 6

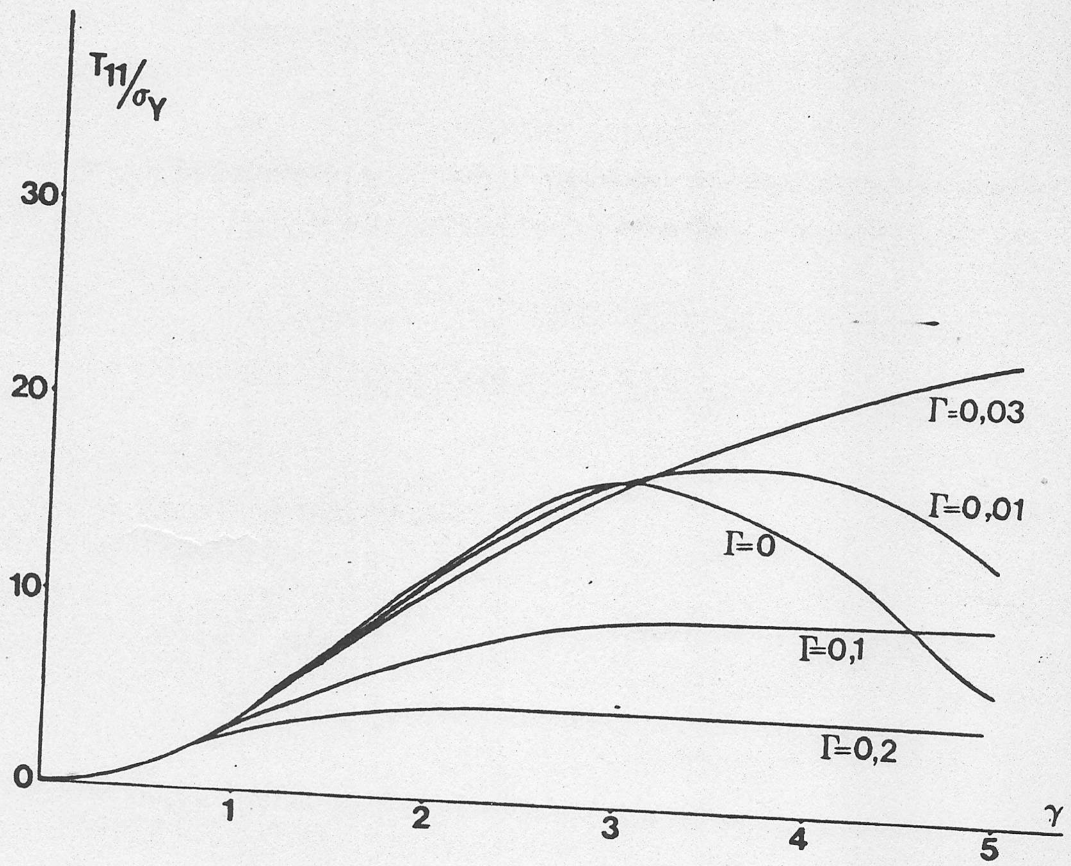


Fig. 7

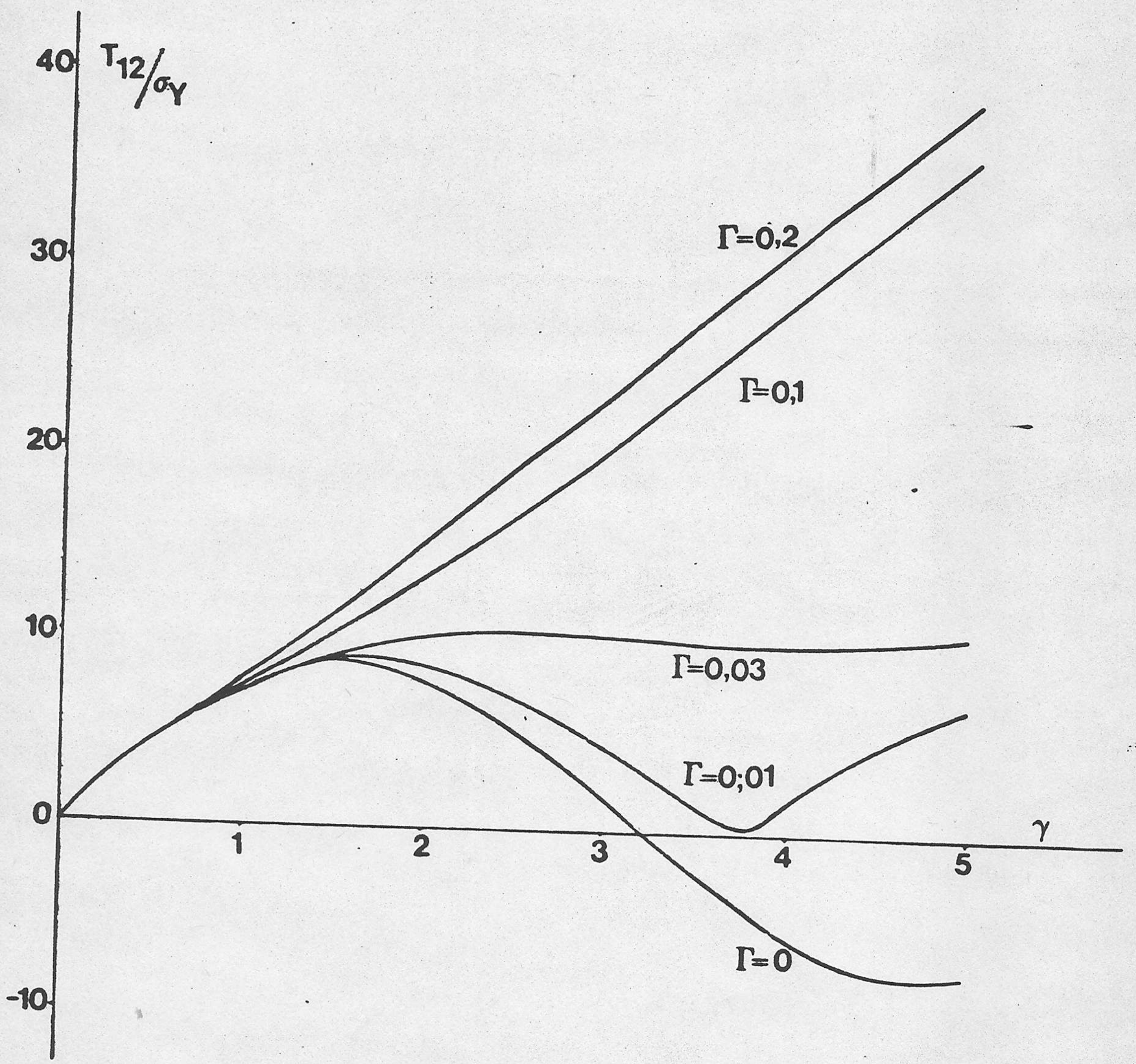


Fig. 8