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BIRATIONAL MODULI AND NONABELIAN COHOMOLOGY, II

by

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0. Introduction

The present paper is a continuation of [1] from which we borrow our ideology, terminology and conventions (with one harmless technical modification, cf. the Remark 1 at the end of this introduction). The aim of [1] was to develop a (nonabelian) cohomological approach to the existence problem of fields of moduli for various algebraic structures such as:

a) polarized finitely presented algebras

b) complete local algebras

c) rigidified algebraic groups

for which the method of Matsusaka and Shimura [6] does not seem to apply. However, our method (as developed there) did not permit us to reobtain the original results of Matsusaka and Shimura on polarized nonsingular projective varieties nor to deal with more global objects (rather than with various kinds of algebras).

In the present paper we fill this gap by further developing our cohomological tool in order to deal with:

d) polarized (possibly singular) projective varieties

e) polarized function fields

f) polarized (non necessary linear) algebraic groups.

Our concepts of polarizations in each of the cases above will be explained in §1 where we also state our main results. Note that in case d) our polarizations are "inhomogenous" . In case e) we get our results only for function fields admitting minimal models in the sense of Mori's program; so if the "minimal model conjecture" [9] is true, we get a good picture for e) in the case of non-uniruled function fields. As for case f) our polarizations are combinations of the classically defined polarizations of abelian varieties and "rigidifications" of linear algebraic groups as defined in [1].

We close our introduction by making two remarks on terminology.

Remark 1. In [1] we denoted by B the dual of the category of field extensions of some fixed field k. To avoid certain logical difficulties it is convenient to slightly modify this definition of B. We shall fix a field extension $k \subset \Omega$ with Ω algebraically closed and tr. deg. Ω/k uncountable. By an embedded field we will understand any intermediate field K between k and Ω such that tr. deg. Ω/K is uncountable. Now we denote by B the dual of the full subcategory of the category of fields whose objects are the embedded fields. Everything which was said in [1] holds for this new B instead of the old one. But here we have the advantage that for any $K \in B$ we have a canonical way to associate an algebraic closure of it $K_a \in B$ (namely K_a = algebraic closure of K in Ω) and an embedding $K \subset K_a$. This will make things easier at a certain point.

Remark 2. By a "variety over field K" we will always understand a quasi-projective geometrically integral scheme over K.

1. Polarizations. Main result

(1.1) It will be convenient to make an "abstract" preparation on polarizations. So let C be a fibred category over B; recall that C is defined by categories $C_K (K \in B)$, covariant functors $C_u : C_K \longrightarrow C_{K'}$ (for any field homomorphism $u : K \longrightarrow K'$) and isomorphisms $C_{u,v} : C_v \circ C_u \rightarrow C_{vu}$. Recall also that the functor $B \longrightarrow S$ (= category of sets) defined by $K \mapsto C_K/\text{iso}$ will be still denoted by C; it is called the "moduli functor".

By a polarization on C we will understand any "fibred functor" $\pi : C \longrightarrow S$ i.e. the giving of the following data: contravariant functors $\pi_K : C_K \longrightarrow S$ (for all $K \in B$) and morphisms $\pi_u : \pi_K \longrightarrow \pi_{K'} \circ C_u$ (for any field homomorphism $u : K \longrightarrow K'$) such that whenever $v : K' \longrightarrow K''$ is another field homomorphism we have $\pi_{vu} = \pi_{K''}(C_{u,v}) \circ \pi_v(C_u) \circ \pi_u$. For any $A \in C_K$, the elements of $\pi(A) = \pi_K(A)$ will be called polarizations on A; note that the group G(A) = G(A,C) defined in [1] (2.13) acts (on the left) on $\pi(A)$.

Given C and π as above one can define a new fibred category C^{π} as follows. For any $K \in B$ the objects of C^{π}_K are pairs (A, η) with $A \in C_K$, $\eta \in \pi(A)$ while morphisms in C_K, the functors C_u and the isomorphisms $C_{u,v}$ are defined in an obvious way.

(1.2) In [1] we implicitely used polarizations in the above sense. For instance (the fibred groupoid structure of) PAL [1] (2.2) is obtained from the fibred groupoid of finitely presented algebras and the fibred functor π associating to any such K-algebra A the set of finite dimensional linear subspaces P of A for which the natural map $K\langle P \rangle \rightarrow A$ is surjective and has a finitely generated kernel.

(1.3) Another example is provided by the fibred groupoid AHA^r [1], (2.7) which is obtained from the fibred groupoid AHA and the fibred functor π which takes any linear algebraic K-group L into the set of all its rigidifications [1] (2.7).

(1.4) Assume C and π are as in (1.1). We say that π is discrete if π_u is an isomorphism for any field homomorphism $u : K \to K'$ for which K and K' are algebraically closed. In example (1.2) π is not discrete while in example (1.3) it is.

(1.5) A functor $C : B \to S$ is said to have property (μ) (minimality property) if for any universal field $K \in B^u$ and any $\xi \in C(K)$ the set $D(\xi, C)$ of algebraically closed members of $D(\xi, C)$ (i.e. of algebraically closed fields of definition of ξ) has a smallest element (recall that $D(\xi, C)$ does not have in general a smallest element even for very nice C's). Property (μ) should be viewed as a "shadow" of the modular properties discussed in [1]. The following (trivial) lemma indicates its connection with property (d_1) from [1] and with polarizations.

(1.6) LEMMA. Let $C : B \quad S$ be a functor. Then

1) If C has property (d_1) it also has property (μ).

2) If C is the "moduli functor" of some fibred category C and if there exists a discrete polarization π on C such that C^{π} has property (μ) then C itself has property (μ). More precisely for any $K \in B^u$ and $(A, \eta) \in C^{\pi}_K$ we have $D_a((A, \eta), C^{\pi}) = D_a(A, C)$.

Next we introduce the three fibred categories we shall be dealing with in the present paper. For any field K let

PRO_K = groupoid of projective K-varieties

FUF_K = groupoid of function fields over K

AGR_K = groupoid of algebraic groups over K,

and let PRO, FUF, AGR denote the corresponding fibred groupoids over B (and also the corresponding moduli functors $B \to S$).

Note that the objects of FUF_K are the regular finitely generated field extensions of K while base change in FUF is defined by the formula $F \mapsto Q(F \otimes_K K')$ for any field homomorphism $K \to K'$ and any $F \in FUF_K$, where Q denotes "taking quotient field".

We will also consider a remarkable fibred subcategory FUF^m of FUF : for any $K \in B$, FUF^m_K will be the full subcategory of FUF_K whose objects are those function fields F/K such that $F \otimes_K K_a / K_a$ has a Q-factorial (terminal) minimal model in the sense of [9].

In what follows we shall define natural discrete polarizations π on PRO, FUF^m, AGR and prove

(1.7) THEOREM. If char k = 0, the functors PRO^{π}, $FUF^{m,\pi}$, AGR^{π} are coarsely representable by birational sets of finitely generated type (i.e. have property (m) in the terminology of [1] (1.4)). Moreover the functors PRO, FUF^m, AGR have the minimality property (μ).

To prove theorem (1.7) we will prove that PRO^{π}, $FUF^{m,\pi}$, AGR^{π} satisfy the properties $(\omega)(s)(\delta_1)(\delta_2)(d_3)$ from [1] (1.4) and apply Theorem (1.5) from [1] and the Lemma (1.6) above. As in [1] the only non-trivial properties to be checked will be (δ_1) and (δ_2). Note that the assertion on PRO^{π} is essentially due to Matsusaka and Shimura [6].

Finally our assertion on FUF^m having property (μ) can also be deduced using our theory in [2], Chapter 2, § 1.

Now let's consider coarse representability of certain (non-polarized) subfunctors of FUF and AGR. Let FUF^g be the subfunctor of FUF corresponding to function fields of general type (i.e. for which the Kodaira dimension equals the transcendence degree). Moreover let AGR^p be the subfunctor of AGR corresponding to "pure" algebraic groups; here an algebraic group Γ over $K = K_a$ is called pure if it is connected and both Aut(P)/Int(P) and Aut(A) are finite groups, where P is the "reductive part" of the "linear part" L of Γ [1] (2.7) and $A = \Gamma/L$ is the "abelian part" of Γ; if $K = K_a$, Γ is called pure if $\Gamma \otimes K_a$ is so.

(1.8) **THEOREM.** If char k = 0, the functors FUF^g and AGR^p are coarsely representable by some birational sets of finitely generated type.

We now concentrate ourselves on defining polarizations. First we have an abstract prolongation procedure; indeed one can easily prove the following.

(1.9) **LEMMA.** Let C be a fibred category over B, C^a its "restriction" to B^a and $\pi : C^a \longrightarrow S$ a fibred functor. Then there is a unique fibred functor still denoted by $\pi : C \longrightarrow S$ (called the canonical prologation of π) such that for all $K \in B$ and $A \in C_K$ we have

$$\pi(A) = \pi(A_a)^{g(K_a/K)} .$$

where A_a is the image of A via the functor $C_K \longrightarrow C_{K_a}$.

(1.10) Let's define a polarization π on PRO as being the canonical prolongation of $\pi : PRO^a \longrightarrow S$ defined by letting $\pi(X)$ be the set of ample elements in the Neron-Severi group $\widetilde{Pic}(X) = Pic(X)/Pic^{\circ}(X)$. Clearly our π is discrete.

(1.11) Let's define a polarization π on FUF^m. First some terminology. Let K be a field of characteristic zero and F a function field over K. By a model of F we undestand a pair (X, ε) where X is a K-variety and $\varepsilon : K(X) \longrightarrow F$ is a K-isomorphism;

when there is no danger of confusion we simply say that X is a model of F. For $K = K_a$ denote by m(F) the set of $\mathbb{Q}$-factorial minimal models of F; recall that it is conjectured that $m(F) \neq \phi$ whenever F is not uniruled [9]. Note also that in order to avoid logical difficulties we work in a universe such that m(F) is really a set. Now assume $K = K_a$, $F \in FUF_K^m$; we shall define in what follows abelian groups Cl(F), Cl°(F), $\overline{\mathrm{Cl}}$(F). We need several remarks.

Remark 1. (essentially cf. [4]; same proof as in [4] p. 33). Let $(X_i, \varepsilon_i) \in m(F)$, $i = 1,2$ and consider any diagram

(diagram 1)

where $\alpha^* = \varepsilon_1 \varepsilon_2^{-1}$, X_3 is smooth and p_i are projective birational with exceptional loci E_i of pure condimension 1. Then $E_1 = E_2$(call it E). In particular p_1 and p_2 induce isomorphisms $Cl(X_1) \simeq Pic(X_3 \setminus E) \simeq Cl(X_2)$.

Remark 2. With the notations above p_1 and p_2 induce isomorphisms $Pic°(X_1) \simeq Pic°(X_3) \simeq Pic°(X_2)$.

Indeed we may assume (by making a base change) that K is uncountable. Consider the diagram

(diagram 2)

Since α and γ are injective so is p_1^*. To prove that p_1^* is also surjective it is sufficient to prove that $M = \mathrm{coker}\ p_1^*$ has countable rank (as an abelian group). But this follows from the fact that $\alpha, \beta, \gamma, \delta$ all have kernels and cokernels of countable rank.

Remarks 1 and 2 imply that we have isomorphisms

$Cl(X_1)/Pic°(X_1) \simeq \overline{Pic}(X_3 \setminus E) \simeq Cl(X_2)/Pic°(X_2)$ and $\overline{Pic}(X_3 \setminus E) \simeq \overline{Pic}(X_3)/\sum \mathbb{Z}[E^j]$

where E^j are the irreducible components of E.

Remark 3. The isomorphism $Cl(X_1) \simeq Cl(X_2)$ from remark 1 does not depend on

the choice of X_3, p_1, p_2 but only on (X_1, ε_1), (X_2, ε_2).
As a consequence of remarks 1-3 we way define $Cl(F) = \varinjlim Cl(X)$, $Cl^\circ(X) = \varinjlim Pic^\circ(X)$, $\overline{Cl}(F) = \varinjlim Cl(X)/Pic^\circ(X)$ (where $X \in m(F)$); note that in the limits above all morphisms are isomorphisms.

An element $\lambda \in \overline{Cl}(F)$ will be called ample if there exists $X \in m(F)$ such that the corresponding element $\lambda_X \in Cl(X)/Pic^\circ(X)$ is in $Pic(X)/Pic^\circ(X)$ and is ample. Now define a fibred functor $\Upsilon: FUF^{m,a} \to S$ (and finally take its canonical extension to FUF^m) by letting $\Upsilon(F)$ be the set of ample elements in $\overline{Cl}(F)$. Clearly our Υ above is discrete.

(1.12) Let's define a polarization Υ on AGR as being the canonical extension of $\Upsilon: AGR^a \to S$ defined below. For $K = K_a$ and $\Gamma \in AGR_K$ let L be the largest connected closed linear subgroup of Γ and $A = \Gamma/L$; then put $\Upsilon(\Gamma) = \Upsilon(L) \times \Upsilon(A)$ where

$\Upsilon(A)$ = set of ample elements in $Pic(A)/Pic^\circ(A)$

$\Upsilon(L)$ = set of isomorphism classes of faithful representations of $L/R_u(L)$ (where $R_u(L)$ is the unipotent radical of L). Since $L/R_u(L)$ is reductive it turnus out that Υ defined above is discrete.

2. Cohomology of G-algebraic groups

For technical reasons it is convenient to introduce some elementary definitions related to group actions on schemes.

If C is a category and G is a group, by a left (respectively right) G-object in C we mean a pair consisting of an object A of C and a group homomorphism $G \to Aut_C(A)$ (respectively a homomorphism from the opposite group G^{op} to $Aut_C(A)$) which we denote by $s \mapsto s_A$ ($s \in G$). A morphism $f \in Hom_C(A,B)$ between two G-objects is called a G-morphism if $s_B \circ f = f \circ s_A$ for all $s \in G$.

So we will speak about left G-sets, left G-groups, left G-rings: these are simply left G-objects in the category of sets, groups, rings.
We will also consider right G-schemes (= right G-objects in SCH, the category of schemes); if K is a left G-field then Spec K will be a right G-scheme. By a right G-scheme X over a right G-scheme Y we will mean a G-morphism $X \to Y$ between two right G-schemes. Furthermore by a right G-variety over a left G-field we mean a right G-scheme X over Spec K such that X/K is a variety. Finally by a right G-algebraic group over a left G-field K we mean a right G-scheme Γ over Spec K which is an algebraic K-group such that the multiplication $\mu: \Gamma \times_K \Gamma \to \Gamma$ and the unit $\varepsilon: Spec\ K \to \Gamma$ are G-morphisms (here note that if X and Y are right G-schemes over a right G-scheme Z then $X \times_Z Y$ has a natural structure of right G-scheme; in particular our $\Gamma \times_K \Gamma$ has one).

Of course the main point with our G-varieties X (and G-algebraic groups) is that for $s \in G$ the automorphisms s_X are not "over K", but only "over K^G".

Note that if X is a right G-scheme over a right G-scheme Z then for any right G-scheme Y over Z the set $X(Y) = \mathrm{Hom}_{SCH_Z}(Y,Z)$ has a natural structure of left G-set defined as follows : for $\alpha \in X(Y)$, $s \in G$, put $s_{X(Y)}\alpha = s_X^{-1} \circ \alpha \circ s_Y$. Moreover if $Z = \mathrm{Spec}\, K$ and $X = \Gamma$ is a right G-algebraic group over K then $\Gamma(Y)$ is a left G-group; in particular one can speak about $H^1(G,\Gamma(K))$. Distinguished elements in H^1 will always be denoted by 1. Furthermore if G_1 is a subgroup of G and K_1/K is an extension of left G_1-fields we have a natural map $H^1(G,\Gamma(K)) \to H^1(G_1,\Gamma(K_1))$ compatible with the natural exact sequences relating H° and H^1.

From now on we shall omit the words "left" and "right" when we refer to G-objects; it will be understood that "algebraic" objects (sets, groups, rings) are "left" and "geometric" objects (schemes, varieties, algebraic groups) are "right".

Our main technical result is the following improvement of [1] (3.3) (see [1], § 3 for teminology):

(2.1) THEOREM. Let K be a G-field, Γ a G-algebraic group over K and $\Sigma \subset H^1(G,\Gamma(K))$ a finite subset. Then :

1) There exists a cofininite subgroup G_1 of G and a constrained finitely generated extension of G_1-fields K_1/K such that Σ maps to 1 via the map $H^1(G,\Gamma(K)) \longrightarrow H^1(G_1,\Gamma(K_1))$.

2) If Γ is connected there exists a regular finitely generated extension of G-fields K_1/K such that Σ maps to 1 via the map $H^1(G,\Gamma(K)) \longrightarrow H^1(G,\Gamma(K_1))$.

The theorem above is better than its analogue in [1] for at least two reasons:

1) Γ need not be defined over K^G

2) Γ need not be linear.

Both these features will be essential in what follows. On the other hand it is reasonable to conjecture that if $K = K_a$ then any Γ as in the theorem is defined over $(K^G)_a$ (for Γ linear, this was proved in [1] (6.4) while for Γ an abelian variety this will be observed below, cf (5.2)).

To prove (2.1) we need the following

(2.2) LEMMA. Let K be a G-field, X a G-scheme of finite type over K and $X^{(G)}$ the set of (non-necessary closed) points p of X such that the group $St(p) = \{ s \in G; s_X(p) = p \}$ contains a cofinite subgroup of G. Then for any maximal element p_1 of $X^{(G)}$ the extension of $St(p_1)$-fields $K(p_1)/K$ is constrained (here $K(p_1)$ = residue field at p_1).

Proof. Take $a \in K(p_1)^{St(p_1)}$ as in [1], proof of Theorem (3.3); it is sufficient to prove that a is algebraic over K. Let $G_1 \subset St(p_1)$, G_1 cofinite in G. Now suppose a is transcendental over K, let Y denote the affine line Spec K[a] with its obvious structure of G_1-scheme and let Z be the closure of p_1 in X, which has a naturally induced structure of G_1-scheme of finte type over K. Moreover the element a induces a rational map still denoted by $a : Z \dashrightarrow Y$. Let $\tilde{Z} \subset Z x_K Y$ be its closed graph. Clearly $\tilde{Z}$ is a G_1-subscheme of $Z x_K Y$ and the projections $\varphi : \tilde{Z} \to Z$ and $\psi : \tilde{Z} \to Y$ are G_1-morphisms. Exactly as in [1] loc. cit., Y possesses a closed point m fixed by some cofinite subgroup G_2 of G ($G_2 \subset G_1$) such that $\psi^{-1}(m) \neq \phi$. Then the scheme $\psi^{-1}(m)$ has a natural structure of G_2-scheme. Letting G_3 be the kernel of the representation of G_2 into the permutation group of the set of irreducible components of $\psi^{-1}(m)$ we get that there is a point $q \in \psi^{-1}(m)$ fixed by G_3 hence so will be $\varphi(q)$. Since φ is birational, $\varphi(q)$ is not the generic point of Z, this contradicting the maximality of p_1 in $X^{(G)}$ and we are done.

Proof of (2.1). By induction it is sufficient to assume that Σ consists of one element; let $f : G \to \Gamma(K)$ represent it. We first construct a G-scheme X over K starting from Γ and f as follows. As a scheme, X will be Γ itself while the action $s \mapsto s_X$ of G is defined by the formula

$$s_X = s_\Gamma \circ R_{f(s)}$$

where $R_{f(s)} : \Gamma \to \Gamma$ is the right translation with $f(s) \in \Gamma(K)$ and s_Γ comes from the structure of G-algebraic group of Γ; to check that $(st)_X = t_X \circ s_X$ one is led to check that $R_{s(f(t))} = s_\Gamma^{-1} \circ R_{f(t)} \circ s_\Gamma$ which follows from the commutative diagram

(diagram 3)

Now we claim that if $\alpha_X \in X(Y)$, $\alpha_X : Y \to X$ is any G_1-morphism of G_1-schemes (with $G_1 \subset G$) over K and if we denote by $\alpha_\Gamma : Y \to \Gamma$ its image in $\Gamma(Y)$ and by $f(s)_Y$ the image of f(s) under the map $\Gamma(K) \to \Gamma(Y)$ then

$$f(s)_Y = \alpha_\Gamma^{-1} s_{\Gamma(Y)} \alpha_\Gamma \qquad \text{for all } s \in G_1$$

(equality holding in the group $\Gamma(Y)$). Indeed the formula above is equvalent to

$$R_{f(s)} \circ \alpha_\Gamma = s_\Gamma^{-1} \circ \alpha_\Gamma \circ s_Y$$

i.e. to $s_X \circ \alpha_X = \alpha_X \circ s_Y$ which is simply the condition of f being a G_1-morphism. Now if we take $\alpha_X : \mathrm{Spec}\, K(p_1) \to X$ as above with p_1 a maximal element of $X^{(G)}$ as in

Lemma (2.2) we get statement 1) in our theorem. Finally if Γ is connected, taking α_X : Spec K(X) $\longrightarrow$ X we get statement 2) in our theorem.

3. Splitting projective G-varieties and G-function fields

In this § we prove Theorem (1.7) for PRO$^{\pi}$ and FUF$^{m,\pi}$ and we also prove Theorem (1.8) for FUF^{g}. We start by providing Picard schemes of projective G-varieties with G-actions:

(3.1) LEMMA. Let $K = K_a$ be a G-field and X be a projective G-variety over K. Then $\mathrm{Pic}^{\circ}_{X/K}$ is a G-algebraic groups in a natural way. Moreover Pic(X), Pic°(X), $\overline{\mathrm{Pic}}(X)$ are G-groups and $\pi(X)$ is a G-set (π being the polarization defined in (1.10)).

Proof. For $s \in G$ let σ = Spec s_K be the corresponding automorphism of Spec K, let Spec K^{σ} be Spec K itself viewed as a scheme over Spec K via σ and let $X^{\sigma} = X \times_{\mathrm{Spec}\, K} \mathrm{Spec}\, K^{\sigma}$. Then s_X induces a K-isomorphism $\tilde{s}_X : X \longrightarrow X^{\sigma}$ so we get an induced isomorphism $\tilde{s}_X^{*} : \Gamma^{\sigma} = \mathrm{Pic}^{\circ}_{X^{\sigma}/K^{\sigma}} \longrightarrow \mathrm{Pic}^{\circ}_{X/K} = \Gamma$. Let $s_{\Gamma} : \Gamma \rightarrow \Gamma$ be defined as $s_{\Gamma} = \sigma_X \circ (\tilde{s}_X^{*})^{-1}$ where $\sigma_X : \Gamma^{\sigma} \rightarrow \Gamma$ is the natural projection. One checks that $s \mapsto s_{\Gamma}$ gives the desired structure of G-algebraic group on Γ. Same construction in the remaining cases.

(3.2) LEMMA. Let $K = K_a$ be a G-field of characteristic zero and F a G-function field over K (i.e. a function field which is a G-field extension) with $m(F) \neq \phi$. Then for any $X \in m(F)$, $\mathrm{Pic}^{\circ}_{X/K}$ is a G-algebraic group in a natural way. Moreover Cl(F), Cl°(F), $\overline{\mathrm{Cl}}(F)$ are G-groups and $\pi(F)$ is a G-set (π being defined as in (1.11)).

Proof. Same game as in (3.1) (use Remark 1 in (1.11)).

(3.3) LEMMA. Let K be a G-field of characteristic zero (non necessary algebraically closed), F a G-function field over K and X a model of F such that $X \otimes K_a \in m(F \otimes K_a)$. Then Cl(X) has a natural structure of G-group. Moreover, if $L \in \mathrm{Pic}(X) \cap (\mathrm{Cl}(X))^G$ then $P = P(H^{\circ}(X,L))$ has a natural structure of G-variety over K.

Proof. The G-group structure may be defined by considering a diagram as in Remark 1 from (1.11). If we view elements of Cl(X) as isomorphism classes of reflective sheaves of rank 1 on X then the G-action on Cl(X) may be described as follows. For $s \in G$ let σ, K^{σ}, X^{σ} be as in (3.1); corresponding to s_F we get a rational map $\tilde{s}_X : X \dashrightarrow X^{\sigma}$ hence a diagram

$$X \xleftarrow{i} X_o \xrightarrow{\tilde{s}_X} (X^{\sigma})_o \xrightarrow{j} X^{\sigma} \xrightarrow{p} X$$

where X_o, $(X^{\sigma})_o$ are nonsingular open subsets of X and X^{σ} respectively, whose complements have codimension ≥ 2 and p is the canonical projection. Then $s_{\mathrm{Cl}(X)}$ is defined by $[L] \longmapsto [i_*(\tilde{s}_X)^* j^* p^* L]$. Now if L is invertible and G-invariant there are isomorphisms $\tau_s : L \simeq i_*(\tilde{s}_X)^* j^* p^* L$ for all $s \in G$ (note that τ_s is unique up to multiplication with some nonzero element of K). We get isomorphisms (natural up to scalar multiplication)

$$H^\circ(X,L) \simeq H^\circ(X^\sigma, p^*L) \simeq H^\circ((X^\sigma)_o, j^*p^*L) \simeq H^\circ(X_o, (\tilde{s}_X)^* j^* p^* L) \simeq$$
$$\simeq H^\circ(X, i_*(\tilde{s}_X)^* j^* L) \simeq H^\circ(X,L)$$

These isomorphisms induce a structure of G-variety on $\mathbb{P}$.

(3.4) The following definition will play a key role in what follows (as its algebraic analogue played in [1]). A G-scheme Z (respectively a G-function field F) over a G-field K will be called split if there is a K-isomorphism $\varphi : Z \simeq Z_o \otimes K$ with Z_o a K^G-scheme (respectively a K-isomorphism $\varphi : F \simeq Q(F_o \otimes K)$ with F_o a function field over K^G) such that we have $\varphi \circ s_Z \circ \varphi^{-1} = id \otimes s_K$ (respectively $\varphi \circ s_F \circ \varphi^{-1} = Q(id \otimes s_K)$) for all $s \in G$. A φ as above will be called a splitting of X (respectively of F).

(3.5) LEMMA. Let K be a G-field.

1) Let X_o and Y_o be K^G-schemes and let $X_o \otimes K$ and $Y_o \otimes K$ be given the natural structures of split G-schemes over K. Then any G-morphism between them has the form $\varphi_o \otimes K$ where φ_o is morphism $X_o \longrightarrow Y_o$.

2) Let X be a G-scheme over K and assume we have a covering $\{U_i\}$ of it with open G-invariant subsets such that all intersections $U_{i_1} \cap U_{i_2} \cap \dots \cap U_{i_p}$ $(p \geq 1)$ are affine and split. Then X itself is split.

3) Let X be a split G-scheme and Y a closed G-invariant subscheme X. Then Y with its induced G-scheme structure is split.

Proof. First one proves 1) for X_o, Y_o affine (by just noting that $K[X_o \otimes K]^G = K^G[X_o]$). Then one proves 2) using the affine case of 1) to glue the different splittings of the U_i's. Next one proves 3) for X affine by noting that the ideal $I \subset K[X]$ which defines Y is a $K[G]$-submodule of the split $K[G]$-module $K[X]$ hence by [1] (3.4), I itself is split. Next one proves 1) and 3) in general by reducing to the affine case via 2).

(3.6) LEMMA. Let K be a G-field and Z a projective G-scheme over K. Assume K^G is a field of definition for Z and that $Aut_{Z/K}$ is quasi-compact (equivalently of finite type). Then there exists a cofinite subgroup G_1 of G and a finitely generated constrained extension K_1/K of G_1-fields such that $Z \otimes K_1$ is a split G_1-variety.
If in addition $Aut_{Z/K}$ is connected then the same conclusion holds with $G_1 = G$ and K_1/K "regular" instead of "constrained".

Proof. Let $\varphi : Z \rightsquigarrow Z_o \otimes K$ be an isomorphism, with Z_o a K^G-variety. Then the association

$$s \mapsto f(s) = \varphi \circ s_Z \circ \varphi^{-1} \circ (\mathrm{id} \otimes s_K^{-1}) \in \mathrm{Aut}(Z_o \otimes K/K)$$

defines a class in $H^1(G, \Gamma(K))$ where $\Gamma = \mathrm{Aut}_{Z_o/K^G}$. Now applying Theorem (2.1) to this class we may assume (after replacing G and K by G_1 and K_1) that $f(s) = \alpha^{-1} \circ (\mathrm{id} \otimes s_K) \circ \alpha \circ (\mathrm{id} \otimes s_K^{-1})$ for some K-automorphsm α of $Z_o \otimes K$. Then $\alpha \circ \varphi$ will be a splitting for Z.

Now let's pass to the proof of Theorem (1.7) for PRO and FUF^m and their "polarized" versions. With the terminology of [1] it is sufficient by [1] (1.5) to prove that PRO^{π}, $FUF^{m,\pi}$ have properties (δ_1) and (δ_2). This immediately follows from the statement below:

(3.7) THEOREM. Let K be an algebraically closed G-field. Assume X is a projective G-variety and $\lambda \in \pi(X)^G$ (respectively assume F is a G—function field of characteristic zero over K with $m(F) \neq \phi$ and $\lambda \in \pi(F)^G$). Then there exists a cofinite subgroup G_1 of G, a finitely generated constrained extension K_1/K of G_1-fields, a splitting $\varphi : X \otimes K_1 \rightarrow X_o \otimes K_1$ (respectively a splitting $\varphi : Q(F \otimes K_1) \rightarrow Q(F_o \otimes K_1)$) and a polarization $\lambda_o \in \pi(X_o)$ (respectively $\lambda_o \in \pi(F_o)$) such that the images of λ_o and λ in $\pi(X \otimes K_1)$ (respectively in $\pi(Q(F \otimes K_1))$) are the same. Same statement holds with $G_1 = G$ and K_1/K "regular" instead of "constrained". Moreover one can take λ_o above to be represented by some element in $\mathrm{Pic}(X_o)$.

Proof. We shall consider only the case of G-function fields (the case of projective G-varieties being similar and easier). Assume $\lambda \in \pi(F)^G$ is ample on some X, and let $\tilde{\lambda}$ be the image of λ in $H^1(G, \mathrm{Pic}^\circ(X))$. By Theorem (2.1) one can find a cofinite subgroup G_1 of G and a finitely generated constrained extension K_1/K of G_1-fields such that $r_1(\tilde{\lambda}) = 1$ in the diagram below:

(diagram 4)

where we have put $\Gamma^\circ = \mathrm{Pic}^\circ_{X/K}$, $\Gamma = \mathrm{Cl}(X)$, $\Gamma_1 = \mathrm{Cl}(X_1)$, $X_1 = X \otimes K_1$.
Same statement holds with $G_1 = G$ and K_1/K "regular" instead of "constrained".

A diagram chase shows that $\bar{r}(\lambda) = p_1(x_1)$ for some $x_1 \in Cl(X_1)^{G_1}$. We claim that $x_1 \in Pic(X_1)$ and it is ample; indeed we have a commuatative diagram

(diagram 5)

and by hypothesis $\lambda = p(x)$ with $x \in Pic(X)$ ample so $p_1(x_1) = p_1(r(x))$ hence $x_1 - r(x) \in \Gamma^{\circ}(K_1)$ which implies our claim. Note moreover that the image λ_1 of λ in $\mathcal{T}(Q(F \otimes K_1))$ is well defined and is "represented" by x_1. Let L_1 be a line bundle on X_1 corresponding to x_1 and let $n \geq 1$ be such that both $L_1^{\otimes n}$ and $L_1^{\otimes(n+1)}$ are very ample. By Lemma (3.3) $Z_1 = P(H^{\circ}(X_1, L_1^{\otimes n}))$ and $Z_1' = P(H^{\circ}(X_1, L_1^{\otimes(n+1)}))$ have natural structures of G_1-varieties. Applying Lemma (3.6) we may assume (upon modifying G_1 and K_1) that both Z_1 and Z_1' are split. Let Y_1 and Y_1' be the images of X_1 into Z_1 and Z_1' respectively. By Lemma (3.5) Y_1 and Y_1' are split G_1-varieties hence X_1 is split. Now corresponding to Y_1 and Y_1' we have two splittings $\varphi : X_1 \to X_1^{o} \otimes K_1$ and $\varphi' : X_1 \to (X_1^{o})' \otimes K_1$ and ample line bundles L_n^{o} on X_1^{o} and $(L_{n+1}^{o})'$ on $(X_1^{o})'$ whose inverse images via φ and φ' are $L_1^{\otimes n}$ and $L_1^{\otimes(n+1)}$. By Lemma (3.5) $\varphi' \circ \varphi^{-1} = \varphi_o \otimes K_1$ for some $\varphi_o : X_1^{o} \to (X_1^{o})'$. Put $L_{n+1}^{o} = \varphi_o^{*}((L_{n+1}^{o})')$. Then λ_1 may be represented as the difference in $Cl(X_1)$ of the images of L_{n+1}^{o} and L_n^{o}. Our theorem is proved.

In the next § we shall use a slightly amplified version of (3.7) for projective G-varieties (whose proof is the same), namely:

(3.8) Amplification. Assume in (3.7) that K is not algebraically closed anymore but assume instead that there exists an algebraically closed G-subfield K' of K such that X and λ are deduced via base change from some G-variety X' over K' and some $\lambda' \in \mathcal{T}(X')$. Then the conclusion of (3.7) still holds.

Finally note that to prove Theorem (1.8) it is sufficient to prove the following:

(3.9) THEOREM. Let $K = K_a$ be a G-field of characteristic zero and F a G-function field of general type. Then there exists a cofinite subgroup G_1 of G and a finitely generated constrained extension of G_1-fields K_1/K such that $Q(F \otimes K_1)/K_1$ is a split G_1-function field. Same statement with $G_1 = G$ and K_1/K "regular" instead of "constrained".

Proof. Let X be a smooth projective model of F/K and $R = \bigoplus R_n$, $R_n = H^{\circ}(X, \omega_{X/K}^{\otimes n})$ its canonical ring. One sees immediatly that it has a structure of

G-ring. Choose n such that the n-canonical map φ_n of X is birational onto its image and let R* be the K-subalgebra of R genrated by R_n. Then R* is a polarized K[G]-algebra with polarization given by R_n. We condude by [1], Theorem (4.2).

4. SPLITTING G-ALGEBRAIC GROUPS

In this § we prove Theorem (1.7) for AGR^{π} and Theorem (1.8) for AGR^{p}.

First let s give a corresponding definition for "splittings" : a G-algebraic group Γ will be called split if there exists a K-isomorphism of algebraic groups $\varphi : \Gamma \to \Gamma_o \otimes K$ (Γ_o some K^G-algebraic group) which is a splitting in the sense of (3.4). Our main result is:

(4.1) THEOREM. Let Γ be a G-algebraic group over an algebraically closed G-field K of characteristic zero and let L be the largest linear connected closed subgroup of G and $A = \Gamma/L$ (clearly L and A inherit natural structures of G-algebraic groups). Assume there is a maximal reductive subgroup P of L which is G-invariant and there exists a polarization $(\rho, \lambda) \in \pi(\Gamma)^G$ with ρ coming from a faithful K[G]-representation W of $L/R_u(L)$. Then there exists a cofinite subgroup G_1 of G and a constrained extension K_1/K of G_1-fields such that $\Gamma \otimes K_1$ is a split G_1-algebraic group over K_1. Moreover if $\Gamma \otimes K_1 \longrightarrow \Gamma_o \otimes K_1$ is a splitting, there exists a polarization $(\rho_o, \lambda_o) \in \pi(\Gamma_o)$ such that (ρ_o, λ_o) and (ρ, λ) have the same image in $\pi(\Gamma \otimes K_1)$. Same statement holds with $G_1 = G$ and K_1/K "regular" instead of "constrained".

Proof. We use an approach similar to that in [3]. For simplicity we shall assume in what follows that Γ is connected.

Step 1 (skew equivariant Chevalley construction). By [1], Theorem (1.6) there exists a cofinite subgroup G_2 of G and a finitely generated constrained extension K_2/K of G_2-fields (respectively $G_2 = G$ and K_2/K regular) such that $L \otimes K_2$ is a split G_2-algebraic group and $W \otimes K_2$ is a split $K_2[G_2]$-module. We claim that there exists a K-linear subspace V of K[L] having the following properties:

1) V is G_2-invariant
2) V is L-invariant (L acting via right translations)
3) $(V \cap M)K[L] = M$ (M = ideal of the unit)
4) $\dim_K V < \infty$

Indeed one can find a space E with properties 2), 3), 4). Next note that for any $s \in G$, sE will still have properties 2), 3), 4) [for 2) use the diagram in the proof of (2.1)]. Put $V = \sum sE$ where s runs in G_2; clearly V satisfies properties 1), 2), 3). To check it satisfies also 4) note that

$$\dim_K V = \dim_{K_2}(V \otimes K_2) = \dim_{K_2}(\sum [s(E \otimes K_2)])$$

But the latter number is finite because there exists a finite dimensional $K_2^{G_2}$-subspace E_o of $K_2^{G_2}[L_o]$ (where $L \otimes K_2 \simeq L_o \otimes K_2$) such that $E \otimes K_2 \subset E_o \otimes K_2$. Now let

$d = \dim(V\cap M)$, $P = P(\wedge^d V)$, $p_o = P(\wedge^d (V\cap M))$. There is a naturally induced G_2-actions on P letting p_o fixed. The action map $L\times P \longrightarrow P$, $(b,p)\mapsto bp$ is then a G_2-morphism.

Step 2 (skew equivariant version of [2] p. 96). Recall our construction in [2] p. 96: we start with actions $\tau : L\times(\Gamma\times P) \longrightarrow \Gamma\times P$, $\tau(b,(g,p)) = (gb^{-1}, bp)$ and $\theta : \Gamma\times(\Gamma\times P) \rightarrow \Gamma\times P$, $\theta(x,(g,p)) = (xg,p)$. Both τ and θ are G_2-morphisms. As shown in loc. cit. there is a cartesian diagram

(diagram 6)

with w projective, u a principal bundle for (L,τ) and θ descending to an action $\bar{\theta} : \Gamma\times Z \rightarrow Z$ such that the isotropy of $z_o = u(1,p_o)$ in Γ is the identity. One cheks that Z inherits a structure of G_2-vartiety , u, w, $\bar{\theta}$ are G_2-morphisms and z_o is fixed by G_2. Consequently the immersion $\varphi : \Gamma \rightarrow Z$, $x \longmapsto xz_o$ is a G_2-morphism. The closure $\bar{\Gamma}$ of the image of this immersion in Z will be a G_2-subvariety of Z hence its normalisation $\hat{\Gamma}$ will inherit a structure of G_2-variety. We have a diagram

(diagram 7)

Let $D = \hat{\Gamma}\setminus\hat{\varphi}(\Gamma)$; it has pure codimension 1 because v is affine so we view D as a reduced effective Weil divisor. We agree to put $X_2 = X\otimes K_2$ for any K-scheme X and $u_2 = u\otimes K_2$ for any morphism u of K-schemes. So in particular we have a diagram as above with Γ, v, A, ... replaced by Γ_2, v_2, A_2,... .

Step 3 (Splitting) First by applying (3.8) to (A_2, λ_2) there exist a cofinite subgroup G_1 of G_2, a finitely generated constrained extension K_1/K_2 of G_1-fields (respectively $G_1 = G_2$ and K_1/K_2 regular), a splitting $A_1 \simeq A_o \otimes K_1$ and a line bundle $L_o \in Pic(A_o)$ such that L_o and L (where $L\in Pic(A)$ represents λ) have the same image in $\pi(A_1)$. Since the graph of the multiplication map is a G_1-subscheme of $A_1\times A_1\times A_1$ it is the pull-back of a subscheme of $A_o\times A_o\times A_o$ (by Lemma (3.5)) so A_o is seen to be an abelian K_o-variety, $K_o = K_1^{G_1}$ and the splitting of A_1 is a splitting of G_1-algebraic groups (not only of G_1-varieties). Now choose divisors $H_o^1,...,H_o^m$ in some very ample linear system $|L_o^{\otimes N}|$ such that $H_o^1 \cap \dots \cap H_o^m = \phi$ and let H_1^j be their pull-backs on A_1; clearly H_1^j are fixed by G_1. Now for any multiindex $I = (i_1,\dots,i_r)$ put $H_1^I = H_1^{i_1} + \dots + H_1^{i_r}$; then the open subsets of Γ_1 defined by

$$\Gamma_1^I = v_1^{-1}(A_1 \setminus H_1^I)$$

are G_1-invariant and affine. Consider the Cartier divisors $E_1^I = \hat{w}_1^*(H_1^I)$ on $\hat{\Gamma}$ and for any $n \geq 1$ consider the subspaces of $K_1[\Gamma_1^I]$ defined by

$$W_n^I = \left\{ f \in K_1[\Gamma_1^I] \mid (f)_{\hat{\Gamma}_1} + nE_1^I + nD_1 \geq 0 \right\}$$

Clearly W_n^I are finite dimensional $K_1[G_1]$-submodules of the function field $K_1(\hat{\Gamma}_1)$ and $\bigcup W_n^I = K_1[\Gamma_1^I]$. So there is an integer $n \geq 1$ such that for all I, $K_1[\Gamma_1^I]$ is generated as a K_1-algebra by W_n^I. Applying several times [1] Theorem (4.2), we may assume (upon modifying G_1 and K_1) that $K_1[\Gamma_1^I]$ are split $K_1[G_1]$-algebras. By Lemma (3.5) we conclude that Γ_1 itself is a split G_1-variety. This splitting is automatically a splitting as a G_1-algebraic group (use same reasoning as for A_1). Our theorem is proved.

(4.2) Let's explain how one can deduce Theorem (1.7) (for AGR^{π}) from our Theorem (4.1) above. We must prove that AGR^{π} has properties (δ_1) and (δ_2). Let $K \in B^a$, $\Gamma \in AGR_K$ and $\eta = (\rho, \lambda) \in \pi(\Gamma)$ and denote as usual by L and A the linear part of Γ respectively the complete part $A = \Gamma/L$. We claim that one can define a group G acting on K and a structure of G-algebraic group on Γ such that the following hold:

1) $Im(G \to g(K)) = g(\Gamma, \eta)$ (cf [1] (1.3)),

2) There exists a maximal reductive subgroup P of L which is G-invariant,

3) ρ is represented by some K[G]-representation of $L/R_u(L)$.

Our claim and Theorem (4.1) clearly imply (1.7). On the other hand the claim follows by an argument similar to that in the proof of [1] (6.9).

(4.3) Proof of Theorem (1.8) for AGR^p. It is sufficient to check that AGR^p has properties (d_1) and (d_2). Take $K \in B^a$.

Claim 1. If $L \in AGR_K^p$ is linear there is a polarization $\rho \in \pi(L)$, $\rho : L/R_u(L) \to GL_N(K)$ such that whenever $\sigma \in g(K)$ and $u_\sigma : L \to L^\sigma$ is a K-isomorphism we have $\rho \circ \bar{u}_\sigma \simeq \rho^\sigma$ as representations (i.e. the two terms are equal modulo an interior automorphism of $GL_N(K)$; here $\bar{u}_\sigma : L/R_u(L) \to (L/R_u(L))^\sigma$ is induced by u_σ). This was shown in [1] (6.10) (note that we tacitly assumed there that one can take ρ such that $\rho^\sigma \simeq \rho$ for all $\sigma \in g(K)$; this can be done by choosing our ε there to be such that $\varepsilon^\sigma \simeq \varepsilon$ for all $\sigma \in g(K)$; for instance one can take ε to be the sum of a system S of representatives for the set of isomorphism classes of conjugates of a given faithful representation ε_o. Since ε_o is defined over an algebraic number field, S will be finite).

Claim 2. If $A \in AGR_K^p$ is an abelian variety there is a polarization $\lambda \in \pi(A)$ such that $v_\sigma^*(\lambda^\sigma) = \lambda$ for all $\sigma \in g(K)$ and any isomorphism $v_\sigma : A \to A^\sigma$. Indeed by [7] p. 140, the degree map $\varphi : \pi(A)/Aut(A) \to \mathbb{Z}$, ($\varphi(\lambda)$ = top intersection number of λ) has finite fibers. So if Aut(A) is finite we choose $d \in \mathbb{Z}$ such that $\varphi^{-1}(d) \neq \phi$ and let $\lambda \in \pi(A)$ be the sum in $\overline{Pic}(A)$ of all polarizations of degree d; this λ answers our claim.

Now claims 1 and 2 together with Theorem (1.7) clearly imply Theorem (1.8).

5.FURTHER COMMENTS AND QUESTIONS

It is reasonable to make the following

(5.1) Conjecture. AGR has property (δ_1) (hence by [1], Theorem (1.5) also properties (d_1), (g_1)).

Indeed it follows from [1] that AGR^{lin} (= subfunctor of AGR correspoinding to linear groups) has property (δ_1) if char k = 0. Let's also remark that we have:

(5.2) PROPOSITION. If char k = 0 the functor AGR^{ab}(= subfunctor of AGR corresponding to abelian varieties) has property (δ_1).

Proof. Let $K \in B^a$, $A \in AGR_K^{ab}$, $G = G(A, AGR^{ab})$ (cf [1], (2.13)). We want to prove that A is defined over $(K^G)_a$. To see this we construct a subgroup H of G such that $Im(H \to g(K))$ contains a cofinite subgroup of $Im(G \to g(K))$ and such that there exists a polarization $\lambda \in \pi(A)^H$. If this is done one can split the H-variety A as in (3.7) and we are done. Let $\pi^d(A)$ be the set of polarizations in $\pi(A)$ of degree d, pick a $d \geq 1$ such that $\pi^d(A) \neq \phi$, note that G acts on the set $\Sigma = \pi^d(A)/Aut(A)$ and put $H_1 = ker(G \to Aut(\Sigma))$. Hence H_1 is normal of finite index in G (Σ being finite by [7] p. 140). Now pick any $\lambda \in \pi^d(A)$ and let H be the subgroup of G consisting of those $s \in G$ which fix λ ; then $Im(H_1 \to g(K)) \subset Im(H \to g(K))$ and we are done.

(5.3) Using [8] instead of [7] p. 140 one can check that the subfunctor FUF^{K3} of FUF corresponding to K3-surfaces has property (δ_1) and hence properties (d_1), (g_1). It is reasonable to conjecture that the subfunctor FUF^2 of FUF corresponding to function fields of transcendence degree 2 has property (δ_1). In any case its analogue FUF^1 is even coarsely representable by results of Matsusaka and Shimura.

REFERENCES

[1] **A. Buium**, Birational moduli and nonabelian cohomology, I, Preprint INCREST, 34/1987.

[2] **A. Buium**, Differential Function Fields and Moduli of Algebraic Varieties, L.N. 1226, Springer 1986.

[3] **A. Buium**, The automorphism group of a non-linear algebraic group, to appear.

[4] **M. Hanamura**, On the birational automorphism groups of algebraic varieties, Compositio Math. 63, 1 (1987), 123-143

[5] **J.E. Humphreys**, Linear Algebraic Groups, Springer 1975

[6] **T. Matsusaka**, Algebraic deformations of polarized varieties, Nagoya Math. J. 31 (1986), 185-245.

[7] **J.S. Milne**, Abelian Varieties, in: Arithmetic Geometry, Springer 1986.

[8] **H. Sterk**, Finitness results for K3 surfaces, Math. Z. 198 (1985), 507-513.

[9] **P.M.H. Wilson**, Towards birational classification of algebraic varieties, Bull. London Math. Soc. 19 (1987), 1-48.

diagram 1.

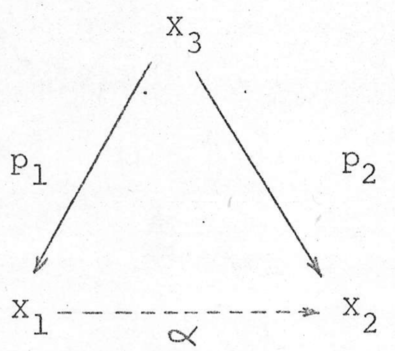

diagram 2.

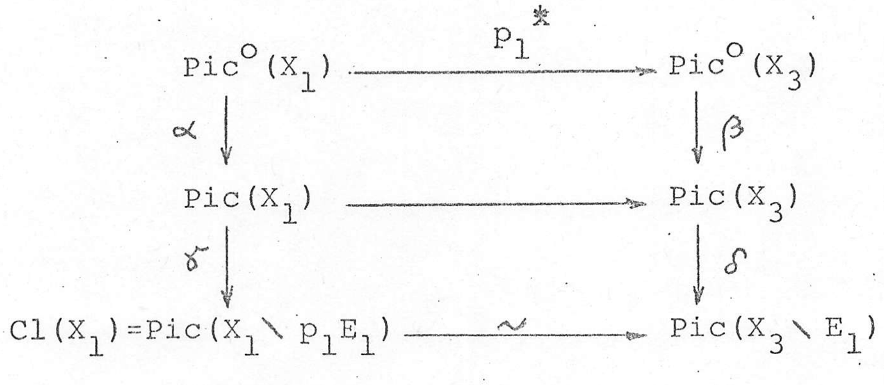

diagram 3.

$$
\begin{array}{ccccc}
\Gamma & \xrightarrow{1\times f(t)} & \Gamma\times\Gamma & \xrightarrow{\mu} & \Gamma \\
\Big\uparrow{\scriptstyle s_{\Gamma}} & & \Big\uparrow{\scriptstyle s_{\Gamma\times\Gamma}} & & \Big\uparrow{\scriptstyle s_{\Gamma}} \\
\Gamma & \xrightarrow[1\times s(f(t))]{} & \Gamma\times\Gamma & \xrightarrow[\mu]{} & \Gamma
\end{array}
$$

diagram 4.

$$
\begin{array}{ccccc}
\Gamma^{G} & \longrightarrow & (\Gamma/\Gamma^{o}(K))^{G} & \longrightarrow & H^{1}(G,\Gamma^{o}(K)) \\
\Big\downarrow{\scriptstyle r} & & \Big\downarrow{\scriptstyle \bar{r}} & & \Big\downarrow{\scriptstyle r_1} \\
\Gamma_1^{G_1} & \xrightarrow[p_1]{} & (\Gamma_1/\Gamma^{o}(K_1))^{G_1} & \longrightarrow & H^{1}(G_1,\Gamma^{o}(K_1))
\end{array}
$$

diagram 5.

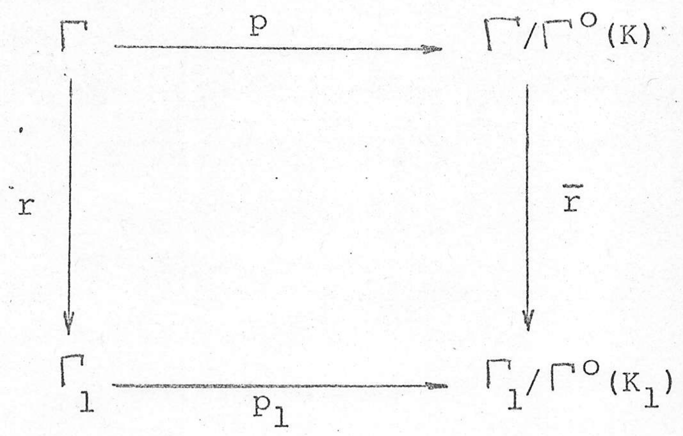

diagram 6.

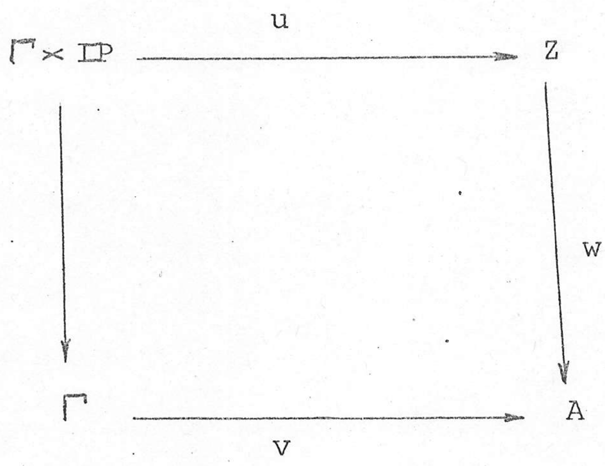

diagram 7.