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ENDOMORPHISMS OF CERTAIN C^*-ALGEBRAS

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INTRODUCTION

Given $T \in L(H)$, where H is a separable infinite dimensional Hilbert space, it is easy to see that we can find an increasing sequence of finite dimensional subspaces H_n with $H=\overline{\cup H_n}$ and $T(H_n) \subset H_{n+1}$ for every n.

Given an unital AF-algebra A, it is natural to consider endomorphisms α for which there exists an increasing sequence of finite dimensional C^*-subalgebras A_n such that $A=\overline{\cup A_n}$ and $\alpha(A_n) \subset A_{n+1}$ for every n. Such an α will be called standard.

In his paper [9], Voiculescu shows that the almost inductive limit automorphisms of an AF-algebra (a notion analogous to that of quasitriangular operator) are approximable by inductive limit automorphisms.

In this note we prove that every endomorphism of an unital AF-algebra can be approximated by standard endomorphisms.

More generally, we can consider $A=\varinjlim(A_n, \varphi_n)$ with A_n in a certain class of C^*-algebras such that $\varphi_n : A_n \to A_{n+1}$ can be classified in some sense. We are interested to understand $End_1(A)$, the set of injective unital $*$-endomorphisms of A, in this case. We can define for A and $\alpha \in End_1(A)$ a new C^*-algebra which contains A and reflects some properties of α.

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§1°. THE GENERAL CASE

Let $(A_n, \varphi_n)_{n\geq 1}$ be an inductive system with A_n unital C^*-algebras and φ_n unital $*$-monomorphisms. Denote by A the C^*-inductive limit of this system, by $i_n : A_n \longrightarrow A$ the canonical injections and by $\mathrm{End}_1(A)$ the set of injective unital $*$-endomorphisms of A.

1.1. DEFINITION. $\alpha \in \mathrm{End}_1(A)$ is called standard with respect to the system (A_n, φ_n) if, eventually after passing to a subsequence and changing notation, we have $\alpha(i_n(A_n)) \subset i_{n+1}(A_{n+1})$. $\alpha \in \mathrm{End}_1(A)$ is called standard if it is standard with respect to some system (A_n, φ_n) with $A = \varinjlim(A_n, \varphi_n)$.

1.2. REMARK. α preserves $\bigcup_n i_n(A_n)$ and we obtain the following commutative diagram

$$\begin{array}{ccc} A_n & \xrightarrow{\varphi_n} & A_{n+1} \\ \alpha_n \downarrow & & \downarrow \alpha_{n+1} \\ A_{n+1} & \xrightarrow{\varphi_{n+1}} & A_{n+2} \end{array}$$

where $\alpha_n := i_{n+1}^{-1} \circ \alpha \circ i_n : A_n \longrightarrow A_{n+1}, n \geq 1$. In this way we obtain a new inductive system (A_n, α_n) and we can consider its C^*-inductive limit.

1.3. REMARK. If we take $B := \varinjlim(A_n, \alpha_n)$, the sequence (φ_n) determines $\varphi \in \mathrm{End}_1(B)$. We can consider the following commutative diagram

$$\begin{array}{ccccccccc} A_1 & \xrightarrow{\varphi_1} & A_2 & \xrightarrow{\varphi_2} & A_3 & \longrightarrow & \cdots & & A \\ \alpha_1 \downarrow & & \alpha_2 \downarrow & & \alpha_3 \downarrow & & & & \downarrow \alpha \\ A_2 & \xrightarrow{\varphi_2} & A_3 & \xrightarrow{\varphi_3} & A_4 & \longrightarrow & \cdots & & A \\ \alpha_2 \downarrow & & \alpha_3 \downarrow & & \alpha_4 \downarrow & & & & \downarrow \alpha \\ A_3 & \xrightarrow{\varphi_3} & A_4 & \xrightarrow{\varphi_4} & A_5 & \longrightarrow & \cdots & & A \\ \downarrow & & \downarrow & & \downarrow & & & & \downarrow \\ \vdots & & \vdots & & \vdots & & & & \vdots \\ B & \xrightarrow[\varphi]{} & B & \xrightarrow[\varphi]{} & B & \xrightarrow[\varphi]{} & \cdots & & \end{array}$$

Denoting by $A(\alpha):=\varinjlim(A,\alpha)$ and $B(\varphi):=\varinjlim(B,\varphi)$ we remark that $A(\alpha)=B(\varphi)$ and A,B can be embedded in $A(\alpha)$. In fact $A(\alpha)=B(\varphi)=\lim(A_n,\beta_n)$ such that (β_n) is an union of a subsequence of (φ_n) and a subsequence of (α_n).

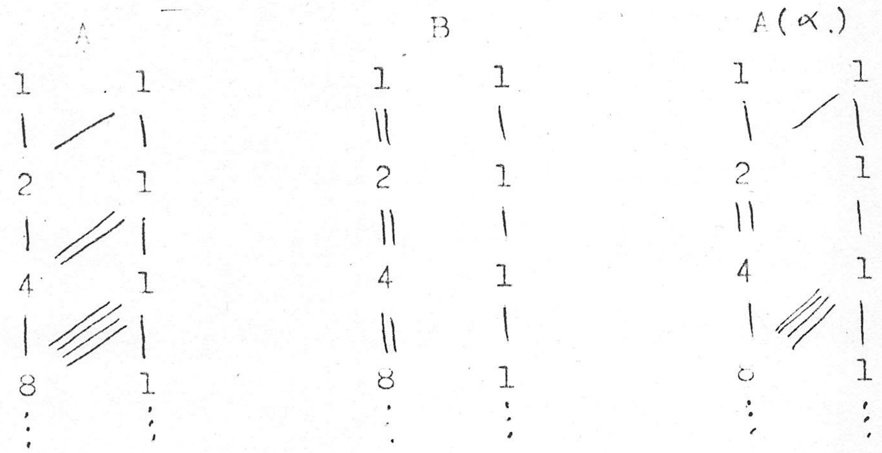
We see also that with the sequence (A_n) and with the bonding maps (φ_n) and (α_n) we can obtain three C^*-algebras: A, B and $A(\alpha)$ which can be different. Moreover, $A(\alpha)$ depends not on the sequence (A_n).

Let's consider the following particular exemples.

1.4. EXEMPLE. Let $A_n=M_{2^{n-1}}\oplus M_1$,

$$\varphi_n=\begin{pmatrix} 1 & 2^{n-1} \\ 0 & 1 \end{pmatrix}, \quad \alpha_n=\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

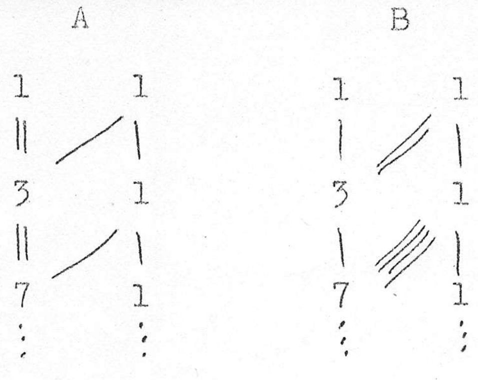
We have $A=\varinjlim(A_n,\varphi_n)=\mathcal{K}^{\sim}$ (the compact operators with adjoined unity), $B=UHF(2^{\infty})\oplus M_1$, $A(\alpha)=B(\varphi)=(UHF(2^{\infty})\otimes\mathcal{K})^{\sim}$. For A, B and $A(\alpha)$ we can take the following Bratteli diagrams



1.5. EXEMPLE. If α is automorphism, we have $A(\alpha)=A$, but A and B can be different even if α is automorphism. Indeed, take

$$A_n=M_{2^n-1}\oplus M_1, \quad \varphi_n=\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad \alpha_n=\begin{pmatrix} 1 & 2^n \\ 0 & 1 \end{pmatrix}.$$

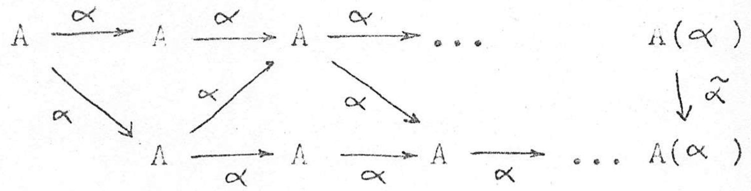
We have $A=(UHF(2^{\infty})\otimes\mathcal{K})^{\sim}$ and $B=\mathcal{K}^{\sim}$:



In order to see that (α_n) induces an automorphism we remark that $K_0(A)=\mathbb{Z}\left[\frac{1}{2}\right]\oplus\mathbb{Z}$ with the lexicographic order and

$$K_0(\alpha)=\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

1.6.REMARK. $\alpha\in \mathrm{End}_1(A)$ induces an automorphism of $A(\alpha)$ by the following diagram



In this way to the pair (A,α) we have associated a greather C^*-algebra in which α becomes automorphism.

Of course, the same construction can be considered in other categories. As an illustrative exemple take the group $\mathbb{Z}$ with the multiplication by 2. We have $\varinjlim(\mathbb{Z},2)=\mathbb{Z}\left[\frac{1}{2}\right]$ and $\mathbb{Z}\left[\frac{1}{2}\right]\xrightarrow{2}\mathbb{Z}\left[\frac{1}{2}\right]$ is an automorphism.

In the case of commutative C^*-algebras, to a pair (X,f) with X a compact Hausdorff space and $f:X\longrightarrow X$ a surjective map we associate $(X(f),\tilde{f})$ where $X(f)$ is another compact Hausdorff space, $\tilde{f}\in \mathrm{Homeo}(X(f))$ and we have a surjective map $X(f)\longrightarrow X$.

§2°. THE CASE OF AF-ALGEBRAS

The case of AF-algebras (A_n finite dimensional) is more tractable because their K_0 groups with order and scale are complete invariants. If we want to know how large is $\mathrm{Aut}(A)$, we study $\mathrm{Aut}(K_0(A),K_0(A)_+,[1_A])$ and use the result that every automorphism of $(K_0(A),K_0(A)_+,[1_A])$ can be lifted to a standard automorphism of A, the error being the composition with an approximately inner automorphism.

It is easy to see that also endomorphisms of $(K_0(A),K_0(A)_+,[1_A])$ can be lifted to standard unital endomorphisms of A.

$$\begin{array}{ccccccccc} K_0(A_1) & \longrightarrow & K_0(A_2) & \longrightarrow & K_0(A_3) & \longrightarrow & \cdots & & K_0(A) \\ & \searrow & & \searrow & & \searrow & & & \downarrow \\ & & K_0(A_2) & \longrightarrow & K_0(A_3) & \longrightarrow & K_0(A_4) & \longrightarrow \cdots & K_0(A). \end{array}$$

In order to prove the following theorem we recall some notations and state some important perturbation results.

2.1. DEFINITION. Let A be an unital AF-algebra. By a nest of A we shall mean an increasing sequence

$$\mathbb{C}\cdot 1=A_0\subset A_1\subset A_2\subset\ldots\subset A_n\subset\ldots$$

with A_n finite dimensional C^*-algebras and $A=\overline{\bigcup_n A_n}$.

If C_1,C_2 are C^*-subalgebras of an arbitrary C^*-algebra C and $\varepsilon>0$ we shall write $C_1\subset^{\varepsilon} C_2$ if

$$\sup\{\inf\{\|x-y\|\mid y\in C_2,\|y\|\leq 1\}\mid x\in C_1,\|x\|\leq 1\}<\varepsilon$$

and $d(C_1,C_2)$ is defined by

$$d(C_1,C_2) = \inf\{\varepsilon>0\mid C_1\subset^{\varepsilon} C_2 \text{ and } C_2\subset^{\varepsilon} C_1\}.$$

2.2. LEMMA. If C_1,C_2 are C^*-subalgebras of C, C_1 is finite dimensional and $\varepsilon>0$, then there is $\delta>0$ depending only on ε and $\dim(C_1)$ such that $C_1\subset^{\delta} C_2\Rightarrow\exists\, u\in\mathcal{U}(C)$ (the unitaries of C) with $C_1\subset \mathrm{Ad}u(C_2)$ and $\|u-1\|<\varepsilon$.

2.3. LEMMA (Christensen). If C_1,C_2 are C^*-subalgebras of C, C_1 is finite dimensional, $0\leq\gamma<10^{-4}$ and $C_1\subset^{\gamma}C_2$, then there is $u\in\mathcal{U}(C)$ such that $C_1\subset\mathrm{Ad}u(C_2)$ and $\|u-1\|<64\,\gamma^{1/2}$.

2.4. REMARK. Let $A=\overline{\bigcup_n A_n}$, $\alpha\in\mathrm{End}_1(A)$ arbitrary and $\varepsilon>0$. Using a complete system of matrix units it is easy to see that there exists an increasing sequence $(n_k)\subset\mathbb{N}$ with $\alpha(A_{n_k})\subset^{\varepsilon/2^k}A_{n_{k+1}}$ for every k. Thus every endomorphism is 'almost' standard.

In his paper, Voiculescu shows that almost inductive limit automorphisms are approximable by inductive limit automorphisms (proposition 2.3. in [9]). In analogy we prove the following result which shows how large is the class of standard endomorphisms.

2.5. THEOREM. Let A be an unital AF-algebra, $\alpha \in \mathrm{End}_1(A)$ and $\varepsilon > 0$. Then for every fixed nest of A there exists $u \in \mathcal{U}(A)$ with $\|u-1\| < \varepsilon$ such that $\mathrm{Ad}u \circ \alpha$ is standard with respect to (a subnest of) the fixed nest.

Proof. Let (B_m) be a nest of A such that $\alpha(B_m) \subset^{\varepsilon_m} B_{m+1}$ with $\varepsilon_m \searrow 0$ (see the remark 2.4.). We shall construct inductively a subnest (A_n) of the nest (B_m), endomorphisms α_n and unitaries u_n such that

$$A_0 = \mathbb{C}\cdot 1,\ A_1 = B_1,\ u_1 = 1,\ \alpha_1 = \alpha$$

$$\alpha_{n+1} = \mathrm{Ad}u_{n+1} \circ \alpha_n,\ \|u_n - 1\| < \varepsilon \cdot 2^{-n} \text{ and } \alpha_n(A_j) \subset A_{j+1},\ 0 \leq j \leq n-1.$$

Having constructed A_n, u_n, α_n, the proof will be concluded since $u = \lim_{n\to\infty} u_n \ldots u_1$ is well defined, $\|u-1\| < \varepsilon$ and $(\mathrm{Ad}u \circ \alpha)(A_n) \subset A_{n+1}$ for every n.

Suppose we have found $A_j = B_{m_j}$, α_j, u_j with the desired properties for $1 \leq j \leq n$. Let

$$\delta_0 = \left(\frac{\varepsilon}{(10^3(n+1))^{n+2}} \right)^{2^{n+1}}$$

and assume $\varepsilon < 10^{-4}$ which is no loss of generality. Since

$\alpha(B_{m_n}) \subset^{\varepsilon_{m_n+p}} B_{m_n+p+1}$ for every $p \geq 0$ and

$$\lim_{p\to\infty} d(\mathrm{Ad}u_n(B_{m_n+p}), B_{m_n+p}) = 0,$$

we can find $m_{n+1} > m_n$ with $\alpha_n(B_{m_n}) \subset^{\delta_0} B_{m_{n+1}}$, therefore

$$\alpha_n(A_n) \subset^{\delta_0} A_{n+1} \text{ with } A_{n+1} := B_{m_{n+1}}.$$

By lemma 2.3. there exists an unitary $v_0 \in A$ with

$$(\mathrm{Ad}v_0 \circ \alpha_n)(A_n) \subset A_{n+1} \text{ and}$$

$$\|v_0 - 1\| < \delta_1 := 2(n+1)10^2 \delta_0^{1/2}.$$

Putting $\gamma_j := (2(n+1)10^2)^j \gamma_o^{1/2}$, we shall find inductively unitaries $v_j \in A_{n+2-j}$, $1 \leq j \leq n$ such that

$(Adv_j \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-j}) \subset A_{n+1-j}$ and $\|v_j - 1\| < \gamma_{j+1}$ $(1 \leq j \leq n)$

Indeed, assume we have found v_j for $j \leq k < n$. Then

$$(Adv_k \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-k-1}) \overset{\delta_k}{\subset} A_{n-k}, \text{where}$$

$$\delta_k \leq 2\|v_k \ldots v_o - 1\| \leq 2(\gamma_1 + \ldots + \gamma_{k+1}) \leq 2(n+1)\gamma_{k+1}.$$

We have

$$(Adv_k \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-k-1}) \subset (Adv_k \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-k}) \subset A_{n+1-k},$$

therefore there is a $v_{k+1} \in \mathcal{U}(A_{n+1-k})$ with

$$(Adv_{k+1} \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-k-1}) \subset A_{n-k}$$

and

$$\|v_{k+1} - 1\| < 10^2(2(n+1)\gamma_{k+1})^{1/2} \leq \gamma_{k+2},$$

which concludes the proof of the existence of the v_j's $(1 \leq j \leq n)$.

Defining $u_{n+1} := v_n \ldots v_o$ we have

$$\|u_{n+1} - 1\| \leq \gamma_1 + \ldots + \gamma_{n+1} \leq (n+1)\gamma_{n+1} < \varepsilon \cdot 2^{-n}$$

and for $0 \leq j \leq n$

$$(Adu_{n+1} \circ \alpha_n)(A_{n-j}) = (Adv_n \circ \ldots \circ Adv_{j+1})(Adv_j \circ \ldots \circ Adv_o \circ \alpha_n)(A_{n-j}) \subset$$
$$\subset (Adv_n \circ \ldots \circ Adv_{j+1})(A_{n+1-j}) = A_{n+1-j}.$$

2.6. COROLLARY. Every $\alpha \in End_1(A)$ for A an unital AF-algebra is of the form $Adv \circ \beta$, where $v \in \mathcal{U}(A)$ and β is standard.

§3°. OTHER CASES

If we want a result similar to the theorem 2.5. for $B = \varinjlim B_n$ with $B_n = C(S^1) \otimes A_n$ and A_n finite dimensional we need a result similar to the lemma 2.3. of Christensen. The difficulty is that closely unitaries are not conjugated.

Elsewhere we have the following lemma which uses functional calculus.

3.1. LEMMA. Let C an unital C^*-algebra and $\varphi : C(S^1) \otimes A_1 \longrightarrow C$ an unital $*$-homomorphism, where A_1 is finite dimensional. Denote by G a set of generators for $C(S^1) \otimes A_1$ (the unitary z for $C(S^1)$

and a complete system of matrix units for A_1). Then for every $\varepsilon > 0$ there exists $\delta > 0$ such that whenever C_1 is a C^*-subalgebra of C with $G \subset^\delta C_1$ there exists an unital $*$-homomorphism $\psi : C(S^1) \otimes A_1 \to C_1$ such that the diagram

$$\begin{array}{ccc} C(S^1)\otimes A_1 & \xrightarrow{\varphi} & C \\ & \psi\searrow \quad \nearrow & \\ & C_1 & \end{array}$$

is approximately commutative to within ε i.e. $\|\psi - \varphi\| < \varepsilon$.

We recall a result concerning the unital $*$-homomorphisms $C(S^1)\otimes A_1 \to C(S^1)\otimes A_2$ with A_1, A_2 finite dimensional C^*-algebras.

3.2.THEOREM. Given an unital $*$-homomorphism

$$\varphi : C(S^1)\otimes(M_{k_1}\oplus \ldots \oplus M_{k_p}) \to C(S^1)\otimes(M_{n_1}\oplus \ldots \oplus M_{n_q})$$

there are a $q\times p$ matrix (m_{ij}) with m_{ij} positive integers (some multiplicities), a $q\times p$ matrix (d_{ij}) with $d_{ij}\in \mathbb{Z}$ (some degrees) such that $\sum_j m_{ij}k_j = n_i$, $i=1,\ldots,q$ and $m_{ij}=0$ implies $d_{ij}=0$ and an unitary $u \in C(S^1)\otimes(M_{n_1}\oplus \ldots \oplus M_{n_q})$ such that $\mathrm{Ad}\,u\circ\varphi$ is homotopic to the (canonical) $*$-homomorphism ψ given by

$$\psi(f_1 \;\ldots\; f_p)(z) = \bigoplus_i \Big(\bigoplus_j \psi_{d_{ij}}(f_j)\Big)$$

where $f_j \in C(S^1)\otimes M_{k_j}$ and

$$\psi_{d_{ij}}(f_j)(z) = \begin{pmatrix} f_j(z^{d_{ij}}) & & & 0 \\ & f_j(1) & & \\ & & \ddots & \\ 0 & & & f_j(1) \end{pmatrix}$$

and $\dim \psi_{d_{ij}}(f_j) = k_j m_{ij}$.

The matrices (m_{ij}) and (d_{ij}) are unique with these properties. In fact $(m_{ij}) = K_0(\varphi)$, $(d_{ij}) = K_1(\varphi)$.

Proof. [4]

3.3.REMARK. In the case of AF-algebras the bonding maps are determined within unitary equivalence by a matrix of positive integers. We can consider inductive systems (B_n, φ_n) where $B_n = C(S^1)\otimes A_n$, A_n finite dimensional and φ_n canonical given by two matrices (multiplicities and degrees). The composition of two canonical maps is not canonical but it is homotopic to a canonical one. To see that, we make some calculations. Let

$$C(S^1)\otimes(M_2\oplus M_1)\xrightarrow{\varphi_1}C(S^1)\otimes(M_3\oplus M_4)\xrightarrow{\varphi_2}C(S^1)\otimes(M_7\oplus M_4)$$

where

$$\varphi_1(f_1\oplus f_2)(z)=\begin{pmatrix}f_1(1)&\\&f_2(z^{-1})\end{pmatrix}\oplus\begin{pmatrix}f_1(z^2)&&\\&f_2(z^3)&\\&&f_2(1)\end{pmatrix}$$

$$\varphi_2(g_1\oplus g_2)(z)=\begin{pmatrix}g_1(z^{-3})&\\&g_2(z)\end{pmatrix}\oplus\ g_2(z^{-2}).$$

We have

$$(\varphi_2\circ\varphi_1)(f_1\oplus f_2)(z)=\begin{pmatrix}f_1(1)&&&&\\&f_2(z^3)&&&\\&&f_1(z^2)&&\\&&&f_2(z^3)&\\&&&&f_2(1)\end{pmatrix}\oplus\begin{pmatrix}f_1(z^{-4})&&\\&f_2(z^{-6})&\\&&f_2(1)\end{pmatrix}$$

Remark that φ_1 is given by $\begin{pmatrix}1&1\\1&2\end{pmatrix},\begin{pmatrix}0&-1\\2&3\end{pmatrix}$ and φ_2 by $\begin{pmatrix}1&1\\0&1\end{pmatrix},\begin{pmatrix}-3&1\\0&-2\end{pmatrix}$. We have

$$\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&1\\1&2\end{pmatrix}=\begin{pmatrix}2&3\\1&2\end{pmatrix},\quad\begin{pmatrix}-3&1\\0&-2\end{pmatrix}\begin{pmatrix}0&-1\\2&3\end{pmatrix}=\begin{pmatrix}2&6\\-4&-6\end{pmatrix}$$

So $\varphi_2\circ\varphi_1\sim\varphi$ where

$$\varphi(f_1\oplus f_2)(z)=\begin{pmatrix}f_1(z^2)&&&&\\&f_1(1)&&&\\&&f_2(z^6)&&\\&&&f_2(1)&\\&&&&f_2(1)\end{pmatrix}\oplus\begin{pmatrix}f_1(z^{-4})&&\\&f_2(z^{-6})&\\&&f_2(1)\end{pmatrix}.$$

Therefore we can only hope that $\alpha\in \mathrm{End}_1(B)$ is homotopic to a standard endomorphism.

3.4. EXEMPLE. Let $(k_n)\subset\mathbb{N}^*$ with $k_n|k_{n+1}$ and $(p_n)\subset\mathbb{Z}$, let $B_n=C(S^1)\otimes M_{k_n}$ and $\varphi_n:B_n\longrightarrow B_{n+1}$ given by

$$\varphi_n(f)(z)=\begin{pmatrix}f(z^{p_n})&&\\&f(1)&\\&&\ddots\;f(1)\end{pmatrix}$$

Consider $(q_n) \subset \mathbb{Z}$ and $\alpha_n : B_n \longrightarrow B_{n+1}$ given by

$$\alpha_n(f)(z) = \begin{pmatrix} f(z^{q_n}) & & & \\ & f(1) & & \\ & & \ddots & \\ & & & f(1) \end{pmatrix}.$$

(α_n) defines $\alpha \in \mathrm{End}_1(B)$ where $B := \varinjlim(B_n, \varphi_n)$. Let $C := \varinjlim(B_n, \alpha_n)$.

Remark that $K_0(B) = K_0(C) = K_0(B(\alpha)) = \{ \frac{m}{k_n} | m \in \mathbb{Z}, n \in \mathbb{N} \}$,

$K_1(B) = \{ \frac{m}{p_1 \cdots p_n} | m \in \mathbb{Z}, n \in \mathbb{N} \}$,

$K_1(C) = \{ \frac{m}{q_1 \cdots q_n} | m \in \mathbb{Z}, n \in \mathbb{N} \}$,

$K_1(B(\alpha)) = \{ \frac{m}{p_1 \cdots p_n q_1 \cdots q_n} | m \in \mathbb{Z}, n \in \mathbb{N} \}$.

Remark that α is automorphism iff (p_n) and (q_n) have the same prime factors.

3.5.REMARK. In certain cases it may be happen that B is AF. In this case we have methods to construct interesting exemples of endomorphisms of AF-algebras. Blackadar has constructed in thies way a symmetry of the CAR algebra for which the fixed points algebra is not AF. In any case, we have $A \subset B$ where $A = \varinjlim(A_n, \varphi_n | A_n)$ and A is AF. We can study the relation between $\mathrm{End}_1(A)$ and $\mathrm{End}_1(B)$.

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