2015 Scientific Report for the project 'Hermitian and quaternionic structures on manifolds and applications'

1 Research papers

The following research papers are updated from 2014 (when they were reported as papers *sent for publication*):

1. Monica Alice Aprodu, Marian Aprodu: *Holomorphic vector bundles on Kahler manifolds and totally geodesic foliations on Euclidean open domains*, **Differential Geometry and its Applications**, 39 (2015), p. 10 – 19;

2. Dmitri V. Alekseevsky, L. David *Tanaka structures (non-holonomic G-structures) and Cartan connections*, **"Conformal and Complex Geometry" in Honour of Paul Gauduchon**, editori: Andrei Moroianu, Liviu Ornea, **Journal of Geometry and Pysics**, vol. 91 (2015), p. 88– 100;

3. Liana David *On cotangent manifolds, complex structures and generalized geometry*, to appear **Annales de l'Institut Fourier**, 28 pages (online publication on the web page of AIF, 25 September 2015).

The following research papers were elaborated during 2015:

1. Paul Baird, Radu Pantilie: *On Ricci solitons and twistorial harmonic morphisms*, sent for publication at **Bulletin de la Societé Mathématique de France**;

2. Liana David, Claus Hertling *Regular F-manifolds: initial conditions and Frobenius metrics*, sent for publication at **Annali della Scuola Normale Superiore di Pisa**.

2 A short description of the research activity

The expected results (the Objectives of the Research) during 2015 are the followings: 1) Totally geodesic foliations; 2) Riemannian submersions; 3) Regular F-manifolds; 4) Harmonic morphisms. The following lines describe the research activity of each member of the team.

Monica Aprodu studied the laplacians on stratifolds and their connection with harmonic maps on Riemann stratifolds. She found an explicit description of the laplacians in some special cases. This represents a step towards the development of a theory of harmonic maps on singular spaces, other than polyhedra. These results will be gathered in a future work (in preparation).

Gabriel Baditoiu studied the problem of the classification of Einstein homogeneous metrics on pseudo-hyperbolic spaces. In previous years, G.B. classified the Lie groups acting effectively and transitively on an n-dimensional non-degenerate hyperboloid (also called a pseudo-hyperbolic space) under the assumptions that (1) G is a closed connected subgroup of $SO_0(n-r, r+1)$ (connected component of the indefinite special orthogonal group) and (2) G acts completely reducible on $\mathbb{R}^{n+1}$; and G.B. showed that any G-homogeneous Einstein pseudo-Riemannian metric on a real, complex or quaternionic pseudo-hyperbolic space, or on a para-complex or para-quaternionic projective space is homothetic to either the canonical metric or the Einstein metric of the canonical variation of a Hopf pseudo-Riemannian submersion. In 2015, for the extension of these results to the case of an actions that is not completely reducible on $\mathbb{R}^{n+1}$, G.B. showed that if G acts effective and transitive on an n-dimensional non-degenerate hyperboloid and under the assumption of (1) but G does not satisfies (2), then G is a semidirect product of two certain Lie groups.

Liana David studied (together with C. Hertling) an important class of F-manifolds, called regular. The notion of an F-manifold was introduced by Herting and Manin and is closely related to the theory of Frobenius manifolds and meromorphic connections. The following results are obtained: an initial condition theorem for regular F-manifolds, which leads to local canonical coordinates on such manifolds. The compatible Frobenius metrics in these coordinates were described. It was proved that, given a regular F-manifold $(M, \circ, e, E)$, any Frobenius metric g on it is uniquely determined, in a neighbourhood of a point $p \in M$, by its value g_p at p together with the endomorphism $X_p \to \nabla_{X_p} E$ of T_pM (where ∇ is the Levi-Civita connection of g), and conversey, starting from a (non-degenerate, $\circ$-invariant) metric

and a well-chosen endomorphism of T_pM, one obtains a (unique) Frobenius metric in a neighbourhood of p. This generalizes Dubrovin's work, who studied Frobenius metrics on semisimple (regular) F-manifolds. Using the initial condition theorem for regular F-manifolds it was also proved that any such manifold can be locally realized as the parameter space of a meromorphic connection in Birkhoff normal form.

Radu Pantilie studied (jointly with P. Baird) the soliton flow on the domain of a twistorial harmonic morphism. For example, the following results are obtained, assuming real-analiticity:

• for the harmonic morphisms given by the Gibbons–Hawking construction, any soliton flow is uniquely determined by its restriction to any local section of the corresponding harmonic morphism,

• for the harmonic morphisms given by the Beltrami fields construction, a contour integral is identified whose vanishing characterises the trivial soliton flows (that is, the Einstein metrics).

3 Workshops, conferences and seminars

In 2015 we report the following activities:

1. Monica Aprodu participated at *The 8th Congress of Romanian Mathematicians*, University Al. I. Cuza, Iasi, June 2015 and at *Workshop for Young Researchers in Mathematics*, Ovidius Constanta University, May 2015;

2. Gabriel Baditoiu participated at the workshop *The interrelation between mathematical physics, number theory and noncommutative geometry*, Erwin Schrdinger International Institute for Mathematical Physics, Viena, March 2015;

3. Liana David participated and gave a 45 minutes talk at *Workshop on Geometry and PDE's*, West University of Timisoara, May 2015, with the title *Unfoldings for regular F-manifolds*;

4. Liana David participated and gave a one hour talk at *Research Seminar, University of Hamburg*, November 2015, with the title *Regular F-manifolds: initial conditions and Frobenius metrics*;

5. Radu Pantilie participated and gave a one hour talk at *Harmonic maps, biharmonic maps, harmonic morphisms and related topics*, University of Cagliari, Italy, June 2015, with the title *(Pluri)harmonic morphisms and the Penrose–Ward tranform.*

We mention that the members of the team take part to the Differential Geometry Seminar at I.M.A.R., the host institution of the project.