

Scientific report for the period October - December 2011

The following research directions were approached:

- Laurențiu Leuştean analysed together with Ulrich Kohlenbach, with the help of proof mining techniques, proofs from nonlinear ergodic theory using Banach limits. The existence of Banach limits is obtained by applying the Hahn-Banach theorem to the space ℓ_∞ and it is not covered by the existing logical metatheorems.

Two such results are the nonlinear generalizations of the classical von Neumann mean ergodic theorem, due to Saejung [4] for CAT(0) spaces and Shioji and Takahashi [5] for Banach spaces with uniform Gâteaux differentiable norm.

The aim of our research is to develop a method to eliminate the arguments using Banach limits, obtaining in this way effective finitary versions of the analysed results.

- Marius Buliga continued the study of emergent algebras, key structures in our future research on the finitization of results related to conical groups and dilation structures.
- Marius Buliga, Laurențiu Leuştean and Emanuel Vlad initiated the analysis of a recent result obtained by Breuillard, Green and Tao [1], which presents a qualitative description of approximate groups as being essentially finite-by-nilpotent. This fundamental result is a vast generalization of the famous Freiman theorem [2] from additive combinatorics.

The proof from [1] is based on the fundamental paper of Hrushovski [3], making a parallel between ideas from mathematical logic, more precisely model theory, and topics in finite combinatorics related to the sum-product phenomenon. Our intention is to use proof mining and emergent algebras to give a simple, geometric proof of the main theorem from [1], without using arguments from nonstandard analysis.

1 Conference talks

Laurențiu Leuştean took part in the Workshop “Mathematical Logic: Proof Theory, Constructive Mathematics”, Mathematisches Forschungsinstitut Oberwolfach (MFO), Germany. MFO organizes regularly workshops in all fields of mathematics. The aim of these workshops is to offer 45 - 48 experts, invited by the Institute’s Director, the opportunity to present recent research results,

especially new methods, and to initiate future research projects. The web page of the workshop is: <http://www.mfo.de/occasion/1145/>.

Laurențiu Leuștean gave an invited talk "Recent developments in proof mining". An abstract of this talk will appear in Oberwolfach Reports.

2 Seminar talks

Beginning with September 2011, we initiated a scientific seminar dedicated to the subject of the grant. This is a weekly seminar at IMAR and it is open to all the interested researchers. Emanuel Vlad presented at the seminar, using [6], two fundamental theorems from Minkowski's geometry of numbers, essential instruments in the proof of the Freiman theorem.

References

- [1] E. Breuillard, B. Green, T. Tao, The structure of approximate groups, (2011), <http://arxiv.org/abs/1110.5008>
- [2] G. A. Freiman, Foundations of a structural theory of set addition, American Mathematical Society, Providence, R.I., 1973. Translated from the Russian, Translations of Mathematical Monographs, Vol 37.
- [3] E. Hrushovski, Stable group theory and approximate subgroups, Journal of the American Mathematical Society, 25 (2012), 189-243.
- [4] S. Saejung, Halpern's iterations in CAT(0) spaces, Fixed Point Theory and Applications 2010, Article ID 471781, 13pp..
- [5] N. Shioji, W. Takahashi, Strong convergence of approximated sequences for nonexpansive mappings in Banach spaces, Proceedings of the American Mathematical Society 125 (1997), 3641-3645.
- [6] T. Tao, V. Vu, Additive combinatorics, Cambridge Studies in Advanced Mathematics 105, Cambridge University Press, Cambridge, 2006.

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