Scientific report for the period December 2014 - November 2015

1   Publications

1.1   Published papers

- D. Ivan, L. Leuştean, A rate of asymptotic regularity for the Mann iteration of κ-strict pseudo-contractions, Numerical Functional Analysis and Optimization 36 (2015), 792-798.

- L. Leuştean, A. Nicolae, A note on an ergodic theorem in weakly uniformly convex geodesic spaces, Archiv der Mathematik 105 (2015), 467-477.

1.2   Papers accepted for publication

- L. Leuştean, A. Nicolae, Effective results on nonlinear ergodic averages in $CAT(\kappa)$ spaces, accepted for publication in Ergodic Theory and Dynamical Systems, DOI: 10.1017/etds.2015.31, 2015.

- L. Leuştean, A. Nicolae, A note on an alternative iterative method for nonexpansive mappings, accepted for publication in Journal of Convex Analysis, 2015.

1.3   Preprints and work in progress

- U. Kohlenbach, L. Leuştean, A. Nicolae, Quantitative results on Fejér monotone sequences, arXiv:1412.5563v2 [math.LO], 2015.

- A. Sipoş, Codensity and Stone spaces, arXiv:1409.1370v3 [math.CT], 2015.

- A. Sipoş, Ultraproducts and uniform rates of asymptotic regularity and metastability, 2015.

- L. Leuştean, On quantitative versions of van der Waerden theorem and its generalisations, 2015.

2 Workshops/Conferences/ Scientific seminars organized

- L. Leuştean and A. Sipoş are the organizers of the FMI/IMAR Logic Seminar.

- L. Leuştean was the co-organizer of a special session Logic in Computer Science at The Eighth Congress of Romanian Mathematicians, Alexandru Ioan Cuza University of Iaşi, 26.06 - 01.07 2015.

3 Conference and Seminar Talks

- A. Sipoş, Codensity and Stone spaces, Logic Colloquium 2015, University of Helsinki, 03 - 08.08 2015.

- A. Sipoş, Codensity and Stone spaces, The Eighth Congress of Romanian Mathematicians (Special Session Logic in Computer Science), Alexandru Ioan Cuza University of Iaşi, 26.06 - 01.07 2015.

4 Participations in Summer Schools

- A. Sipoş took part in the Scandinavian Logic Society Summer School in Logic, University of Helsinki, 27 - 31.07 2015.

5 Short description of the results

In the following we give a brief presentations of the results obtained this year.

- L. Leuştean, A. Nicolae, A note on an ergodic theorem in weakly uniformly convex geodesic spaces, Archiv der Mathematik 105 (2015), 467-477.

In this paper, the authors generalize to a class of complete Busemann and weakly uniformly convex geodesic spaces a multiplicative ergodic theorem due to Karlsson and Margulis [10]. The version published in 2015 is a much enhanced version of the one from 2014. Furthermore, a new section is added, providing an example of a space satisfying our conditions which is not uniformly convex in the sense of Karlsson and Margulis.

- L. Leuştean, A. Nicolae, A note on an alternative iterative method for nonexpansive mappings, accepted for publication in Journal of Convex Analysis, 2015.

In this paper, the authors point out that results on the asymptotic behaviour of an alternative iterative method are corollaries of corresponding results on the well-known Halpern iteration. This iterative method was introduced for Banach spaces in [13] as a discretization of an approximating curve considered in [4].

- U. Kohlenbach, L. Leuştean, A. Nicolae, Quantitative results on Fejér monotone sequences, arXiv:1412.5563v2 [math.LO], 2015.

This paper provides in a unified way quantitative forms of strong convergence results for numerous iterative procedures which satisfy a general type of Fejér monotonicity where the convergence uses the compactness of the underlying set. Fejér monotonicity is a key notion employed in the study of many problems in convex optimization and programming, fixed point theory and the study of inverse problems (see e.g. [2, 3, 12]).
These quantitative versions are in the form of explicit rates of metastability in the sense of T. Tao [11]. The approach given in the paper covers examples ranging from the proximal point algorithm for maximal monotone operators to various fixed point iterations (x_n) for firmly nonexpansive, asymptotically nonexpansive, strictly pseudo-contractive and other types of mappings. Many of the results hold for geodesic spaces like W-hyperbolic spaces or UCW-hyperbolic spaces, in particular for the important class of CAT(0)-spaces due to Gromov.

- A. Sipoş, Codensity and Stone spaces, arXiv:1409.1370v3 [math.CT], 2015.

In this paper, the author explicitly computes some categories of topological spaces important for algebraic logic as images of canonical codensity monads. The method was introduced by Kock [6] and applied by Leinster [8] to obtain the category of ultraproducts.

- A. Sipoş, Ultraproducts and uniform rates of asymptotic regularity and metastability, 2015, http://imar.ro/~asipos/avigad.pdf.

This work has as a point departure Avigad and Iovino's results [1] on the application of model-theoretic methods (based on metric ultraproducts) to obtain uniform bounds on rates of metastability. In contrast to the proof mining techniques, these methods do not give any quantitative information on the bounds.
The author shows that one obtains in a similar way uniform rates of asymptotic regularity for the Mann iteration associated to nonexpansive mappings. Such uniform rates are computed using proof mining methods in [7, 9].

- L. Leuştean, On quantitative versions of van der Waerden theorem and its generalisations, 2015.

In this work in progress, the author analyses the proof mining methods used in [5] to obtain a quantitative van der Waerden theorem. The idea of this research is to obtain a more abstract version of these methods, which can be used to get quantitative versions of generalisations of the van der Waerden theorem.

References

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Project Director,

Laurenţiu Leuştean