Locally conformally Kaehler manifolds with large symmetry group
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Locally conformally Kaehler (LCK) manifolds are hermitian manifolds (M, J, g) the fundamental 2-form ω of which satisfies

$$d\omega = \theta \wedge \omega$$

for a *closed* 1-form θ called the Lee form. A LCK manifold with *parallel* Lee form is called Vaisman.

When compact, Vaisman manifolds are mapping tori over the circle, with compact Sasakian fibres.

In complex dimension 2, the whole list of compact Vaisman surfaces was given by F.A. Belgun (Math. Ann., 2000)

No such structure theorem exists for non-Vaisman LCK manifolds, but a lot of results concerning their geometry and topology are known, see the survey by L. Ornea and M. Verbitsky in Contemporary Mathematics 542, 135–150, 2011. In particular, it is known that compact Vaisman manifolds and compact LCK manifolds with automorphic global Kaehler potential on the universal cover can be characterized in terms of holomorphic actions of circles and 1-dimensional complex tori respectively.

Results concerning the transformation groups of a compact LCK manifold also appear in A. Moroianu and L. Ornea, Manuscripta math. 2009, work partially supported by LEA MathMode.

The present project aims to further clarify the structure of the isometry group of a compact LCK manifold and to try a characterization of homogeneous ones. While it is known that homogeneous compact Vaisman manifolds are elliptic fibrations over homogeneous Kaehler manifolds, nothing is known up to now on the general case of a homogeneous LCK manifold.

Both partners in this project have great experience in conformal and Riemannian geometry on complex manifolds and also have a common research experience, see their extensive lists of publications at http://www.math.polytechnique.fr/moroianu/publi.html and http://gta.math.unibuc.ro/pages/pblee.html