Operads in Algebra, Topology and Physics

series of 4-5 talks given by M. Markl, Prague
based on the book *"Operads in Algebra Topology and Physics"*
by Martin Markl, Steve Shnider and Jim Stasheff
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Operads are mathematical devices which describe algebraic structures of many varieties and in various categories. Operads are particularly important/useful in categories with a good notion of 'homotopy' where they play a key role in organizing hierarchies of higher homotopies. Significant examples first appeared in the 1960's though the formal definition and appropriate generality waited for the 1970's. These early occurrences were in algebraic topology in the study of (iterated) loop spaces and their chain algebras. In the 1990's, there was a renaissance and further development of the theory inspired by the discovery of new relationships with graph cohomology, representation theory, algebraic geometry, derived categories, Morse theory, symplectic and contact geometry, combinatorics, knot theory, moduli spaces, cyclic cohomology and, not least, theoretical physics, especially string field theory and deformation quantization. The generalization of quadratic duality (e.g. Lie algebras as dual to commutative algebras) together with the property of Koszulness in an essentially operadic context provided an additional computational tool for studying homotopy properties outside of the topological setting.

The aim of this series of lectures is to exhibit operads as tools for this great variety of applications. We start with reviewing the history (and prehistory) and providing some feeling as to what operads are good for, both in a topological context and a differential graded algebraic context.

In the remaining talks we try to provide access to some of the myriad of results of the 'renaissance of operads' in which operads have proved their worth in contexts quite different from those of their birth. We emphasize algebraic constructions for operads, geometric examples related to configuration spaces and moduli spaces. If time allows, we mention generalizations such as cyclic and modular operads. Such generalizations are motivated by applications to deformation quantization, string field theory, quantum cohomology and Gromov-Witten invariants.

The talks are intended for researchers and students as well as anyone who wishes to get the flavor of operads and their application. We assume only basic knowledge of algebra, topology and algebraic geometry.