

Description of the area of research:

Numerical semigroups are submonoids of $\mathbb{N}$ with finite complement, which is called *the genus*. These semigroups can be organized as a rooted tree, where the number of gaps is the distance to the root $\mathbb{N}$. It has been a question of ongoing interest to understand the structure of this (infinite) tree, e.g. how fast it grows. In our recent work [1] (which was also supported by the GDRI-ECO-Math network) we suggested to filter the semigroups also by their type, another numerical invariant. We proved that for any positive ℓ , $n(g, g - \ell)$, which denotes the number of numerical semigroups with genus g and type $g - \ell$, is constant for $g \gg 0$. Numerical experiments encouraged us to formulate in loc.cit. a number of questions on the stable number $n(g, g - \ell)$, the inner structure of the semigroups with these parameters and how they change with g . We would like to further explore these ideas, and link numerical invariants to the position in the semigroup tree.

Another topic where we would like to make progress is on matroid theory. It has been a longstanding conjecture of White (1980) that the toric ideal of (the base ring of) a matroid is generated by the binomials corresponding to the symmetric exchange relations among the bases of the matroid. That fact has been verified for special families of matroids, but a general strategy is missing. We want to study how the (White) property is transferred (or not) through contructions on matroids.

Bibliography:

[1] J. Chappelton, J. Ramírez Alfonsín, D.I. Stamate, *Numerical semigroup tree: type-representation*, Experimental Mathematics (2026) available online.
doi: 10.1080/10586458.2025.2604786. Preprint arXiv:2507.15006 [math.AC].

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