

CIPRIAN FOIAS AT 80 EDITORIAL INTRODUCTION

This year, Ciprian Foias is turning eighty, and this issue of the journal is dedicated to him on this occasion. Ciprian contributed in many ways to the development of mathematics in Romania. His influence was felt in many ways: through personal mentoring, collaborations, exciting seminars, and contacts with the wider world of mathematics at a time such contact was scarce.

Particularly important was Ciprian's contribution to the management of *Revue Roumaine* and *Studi și cercetări* (now continued as *Mathematical reports*) until 1978. In editing these publications, Ciprian improved their content and established them as hubs of information, especially in the fields of functional analysis and operator theory. He did this using his international contacts and encouraging his colleagues to publish their best work in these journals. Very importantly, Ciprian published some of his most significant work in the pages of *Revue Roumaine*. In appreciation of his contributions to this journal, the editors are presenting in this issue a selection of the many papers Ciprian published in this journal.

We comment here on two of these publications. The first one of these [4] is an application of local spectral theory and decomposability, which Ciprian pioneered. In this paper, he finds a sufficient condition for a singular measure on the unit circle to generate (via the associated singular inner function and model theory) a reflexive Hilbert space operator. It was known (and easy to see) that reflexivity requires the measure to be diffuse, and Ciprian shows that a certain modulus of continuity— $|t \log t|$ to be precise—implies reflexivity. A complete characterisation of reflexivity remained elusive, and it would have been reasonable to assume that continuity of the measure is really sufficient. It turned out in the end that Ciprian's condition was very close to necessary, as shown by Kapustin. Indeed, it is shown in [10] that reflexivity is equivalent to the fact that the measure puts no mass on thin sets (in the sense of Carleson). This demonstrates the power of decomposability in operator theory, and these concepts came to the fore again in the study of invariant subspaces much more recently [5, 6, 7].

Even more spectacular is the contribution in [1], which attacks one of Halmos's famous ten problems [8] in a rather unexpected way. Halmos observed that the first proofs of the existence of invariant subspaces for compact operators used the fact that such operators are very close to actually having an upper triangular matrix in some orthonormal basis. Based on this observation, he introduced the concept of a quasitriangular operator. This is simply an operator which can be written as an arbitrarily small compact perturbation of an upper triangular matrix. Halmos then went on to ask for a proof that quasitriangular operators do have nontrivial invariant subspaces, assuming that existing methods could be adjusted to this situation. There was little progress on this problem, with the exception of [3], where Douglas and Pearcy prove that an operator with negative Fredholm index is *not* quasitriangular. The stunning result of [1] is a converse: if an operator is not quasitriangular, then some translate of the operator has negative Fredholm index. In particular,

nonquasitriangular operators have invariant subspaces of a particularly simple kind because their adjoints have many eigenvalues. This result was further developed by many authors and led to a general study of operators up to approximate similarity. The most important developments in this area are covered in the monograph [9, 2]. It should be said however that the information in the series of six papers including [1] has not been fully exploited, and the reader will still find their study to be a rewarding experience.

Ciprian is continuing his mathematical work with unabated energy, pursuing many projects with more authors than one can easily count. He no longer serves as an editor of this journal, but he started a tradition of rigor and quality which the editors will strive to emulate.

The editors.

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