CORRIGENDUM TO "ON THE EMBEDDING OF LEVI-FLAT HYPERSURFACES IN THE COMPLEX PROJECTIVE PLANE (AND AN APPENDIX WITH LÁSZLÓ LEMPERT)"

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This corrigendum contains a corrected proof of Lemma 4 from the paper "On the embedding of Levi-flat hypersurfaces in the complex projective plane (and an appendix with László Lempert)" published in Rev. Roumaine Math. Pures Appl. 68 (2023), 95–114.

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The purpose of this corrigendum is to correct a computation error in the proof of Lemma 4 of [1]. The results of the paper are unchanged.

LEMMA 1 ([1, Lemma 4]). *There exists a holomorphic vector field Y on $\mathbb{CP}_2$ and $a \in \mathbb{CP}_2$ such that*

$$\|Y(a)\|_{g_{FS}} = \max_{x \in \mathbb{CP}_2} \|Y(x)\|_{g_{FS}}$$

and a is a strict maximum for $\|Y(\cdot)\|_{g_{FS}}$.

Proof. Consider the vector field $\widetilde{Y} = z_0 \frac{\partial}{\partial z_1} + z_0 \frac{\partial}{\partial z_2}$ on $\mathbb{C}^3 \setminus \{0\}$ and let $Y = \pi_*(\widetilde{Y})$, where $\pi : \mathbb{C}^3 \setminus \{0\} \to \mathbb{CP}_2$ is the canonical map. If $z_0 \neq 0$ and $\zeta_1 = z_1/z_0, \zeta_2 = z_2/z_0$ are non-homogeneous coordinates

$$\pi_*(\widetilde{Y}) = \frac{\partial}{\partial \zeta_1} + \frac{\partial}{\partial \zeta_2}.$$

Since for $z_0 \neq 0$

$$g_{FS} = \frac{1 + |\zeta_2|^2}{(1 + |\zeta_1|^2 + |\zeta_2|^2)^2} d\zeta_1 \otimes d\overline{\zeta}_1 + \frac{1 + |\zeta_1|^2}{(1 + |\zeta_1|^2 + |\zeta_2|^2)^2} d\zeta_2 \otimes d\overline{\zeta}_2$$
$$- \frac{\zeta_1 \overline{\zeta}_2}{(1 + |\zeta_1|^2 + |\zeta_2|^2)^2} d\zeta_2 \otimes d\overline{\zeta}_1 - \frac{\overline{\zeta}_1 \zeta_2}{(1 + |\zeta_1|^2 + |\zeta_2|^2)^2} d\zeta_1 \otimes d\overline{\zeta}_2,$$

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we have

$$
\begin{aligned}
\|Y\|_{FS}^2 &= g_{FS}\Big(\frac{\partial}{\partial\zeta_1}+\frac{\partial}{\partial\zeta_2},\frac{\partial}{\partial\zeta_1}+\frac{\partial}{\partial\zeta_2}\Big)\\
&= 2\mathrm{Re}\Big(\frac{1+|\zeta_2|^2}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}+\frac{1+|\zeta_1|^2}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}\Big)\\
&\qquad -2\mathrm{Re}\Big(\frac{\zeta_1\overline{\zeta}_2}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}+\frac{\overline{\zeta}_1\zeta_2}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}\Big)\\
&= 2\frac{2+|\zeta_1|^2+|\zeta_2|^2-2\mathrm{Re}(\zeta_1\overline{\zeta}_2)}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}\\
&\le 2\frac{2+|\zeta_1|^2+|\zeta_2|^2+2|\zeta_1||\zeta_2|}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}\\
&\le 2\frac{2+|\zeta_1|^2+|\zeta_2|^2+|\zeta_1|^2+|\zeta_2|^2}{(1+|\zeta_1|^2+|\zeta_2|^2)^2} = 2\frac{2(1+|\zeta_1|^2+|\zeta_2|^2)}{(1+|\zeta_1|^2+|\zeta_2|^2)^2}\\
&= \frac{4}{(1+|\zeta_1|^2+|\zeta_2|^2)}
\end{aligned}
$$

with equality if and only if $|\zeta_1| = |\zeta_2|$ and $\zeta_1\overline{\zeta}_2 \in \mathbb{R}_-$. So for $\zeta_1 = \zeta_2 = 0$, $\|Y\|_{FS}^2 = 4$ and it follows that

$$\max_{x\in\mathbb{CP}_2}\|Y(x)\|_{g_{FS}} = \|Y([1:0:0])\|_{g_{FS}} = 4$$

and $[1:0:0]$ is a strict maximum for $\|Y(\cdot)\|_{g_{FS}}$. □

REFERENCES

[1] A. Iordan, *On the embedding of Levi-flat hypersurfaces in the complex projective plane (and an appendix with Laszlo Lempert)*. Rev. Roumaine Math. Pures Appl. **68** (2023), *1-2*, 95–114.

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