Scientific report

for the project

PN-II-ID-PCE-2011-3-0533

for the period

December 2014 – December 2015

In this project we investigate of the space of all Alexandrov surfaces. The importance of Alexandrov spaces stems mainly from the generality of the concept, which allows both differentiable and non-differentiable manifolds to be included in the investigation. An important class of Alexandrov surfaces is that of convex surfaces. Part of our research is devoted to them. In many applications, precisely the non-differentiable case, particularly the theory of polyhedra, including the graphs which are their 1-skeleta, is the prevailing one. Thus, the study of graphs essentially appearing in computer networks is also included in our project. Generalized Halin graphs and Toeplitz graphs are among those studied. Moreover, generalized convexity is also investigated. Concerning the space of Alexandrov surfaces, we tackle approximation issues and undertake a generic study (via Baire categories).

In *the period* ***December 2014 – December 2015:***
- four scientific articles have been published, accepted before 2015, in ISI journals, summing up to an *Impact Factor of* **FI = 2.485** and a *Relative Influence Score of* **SRI =4.039;**
- other five scientific articles have been published in ISI journals, summing up to an *Impact Factor of* **FI = 3.335** and a *Relative Influence Score of* **SRI =5.676;**
- two scientific articles have been accepted in ISI journals, summing up to an *Impact Factor of* **FI = 1.253** and a *Relative Influence Score of* **SRI = 2.925**;
- an article has been published in a **BDI** journal, and other in a volume in **Springer** series **Proc. in Math. & Stat.**.
- other two articles have been accepted in a volume in **Springer** series **Proc. in Math. & Stat.**.

All articles are mentioning the grant.

The members of the research team had **3 talks** at **3 international conferences**, in the mentioned period. A part of the obtained results have been discussed at the geometry seminar of the host institute (IMAR).

Other details concerning the scientific activity of the research team, not included in this report, can be found on the grant's web page, „http://imar.ro/~cvilcu/Web_0533.html”. For example, one can find there links for articles (to the journals which published them or to other places for downloading) and for talks (to the corresponding conferences).

Articles published in 2015, accepted before 2015

1. K. Adiprasito, T. Zamfirescu, *Few Alexandrov surfaces are Riemann*, J. Nonlinear Convex Anal., 16 (2015), 1147-1153; **ISI**, FI= 0,655, SRI= 0,939.

We demonstrate that, in most Alexandrov surfaces of curvature bounded below, most points are not interior to any geodesic. Thus, these surfaces are not Riemannian, in contrast to the “almost Riemannian” structure found by Otsu–Shioya in any Alexandrov space.

2. K. Adiprasito, R. Sanyal: *An Alexander-type duality for valuations*, P. Am. Math. Soc., 143 (2015), 833-843; **ISI**, FI= 0,681, SRI= 1,129.

We prove an Alexander-type duality for valuations for certain subcomplexes in the boundary of polyhedra. These strengthen and simplify results of Stanley (1974) and Miller-Reiner (2005). We give a generalization of Brion’s theorem for this relative situation and we discuss the topology of the possible subcomplexes for which the duality relation holds.

3. L. Yuan, T. Zamfirescu: *Right triple convex completion*, J. Convex Analysis, 22 (2015) 291-301; **ISI**, FI= 0,552, SRI= 0,850.

A set M in a Hilbert space is rt-convex if every pair of its points is included in a 3-point subset {x; y; z} of M making a right angle. We find here for various families of sets the minimal number of points necessary to add to the sets in order to render them rt-convex. For example, for convex bodies this number is at most 2.

4. J. Itoh, J. Rouyer, C. Vîlcu: *Moderate smoothness of most Alexandrov surfaces*, Internat. J. Math., 26 (2015) 1540004 (14 pages); **ISI**, FI=0,597, SRI= 1,121.

We show that, in the sense of Baire categories, a typical Alexandrov surface with curvature bounded below by κ has no conical points. We use this result to prove that, on such a surface (unless it is flat), at a typical point, the lower and the upper Gaussian curvatures are equal to κ and ∞ respectively.

5. J. Itoh, J. Rouyer, C. Vîlcu: *On the Theorem of the Three Perpendiculars*, Elem. Math., 70 (2015), 71-78; **BDI**.

We show that the Theorem of the Three Perpendiculars holds in any finit-dimensional space form.

11. F. Nadeem, A. Shabbir, T. Zamfirescu: *Hamiltonian connectedness of Toeplitz graphs*, in: P. Cartier et al. (eds.), *Mathematics in the 21st Century*, Springer Proc. in Math. & Stat., vol. 98 (2015), 135-149.

A Toeplitz graph is a graph with a Toeplitz adjacency matrix. In this paper we investigate the property of hamiltonian connectedness for some undirected Toeplitz graphs.

Other articles published in 2015

1. J. Itoh, **J. Rouyer**, **C. Vîlcu**: *Simple closed geodesics on most Alexandrov surfaces*,
Adv. Math. 278 (2015), 103-120; **ISI**, FI=1,294, SRI= 3,004.
We study the existence of simple closed geodesics on most (in the sense of Baire category) Alexandrov surfaces with curvature bounded below, compact and without boundary. We show that it depends on both the curvature bound and the topology of the surfaces.

2. L. Montejano, **T. Zamfirescu:** When is a Disk Trapped by Four Lines?, *Graphs Combin.* **31** (2015) 467-476; **ISI**, FI=0,388, SRI= 0,706.
We study the problem of determining when a convex disk is trapped by four lines.

3. X. Feng and L. Yuan, **T. Zamfirescu:** *Acute Triangulations of Archimedean Surfaces. The Truncated Tetrahedron*, Bull. Math. Soc. Sci. Math. Roumanie 58 (2015) 271-282; **ISI**, FI=0,521, SRI= 0,5.
We prove that the surface of the regular truncated tetrahedron can be triangulated into 10 non-obtuse geodesic triangles, and also into 12 acute geodesic triangles. Furthermore, we show that both triangulations have minimal size.

4. K. Adiprasito, B. Benedetti: *Tight complexes in 3-space admit perfect discrete Morse functions*, Eur. J. Comb. 45 (2015), 71-84; **ISI**, FI=0,653, SRI= 1,23.
In 1967, Chillingworth proved that all convex simplicial 3-balls are collapsible. Usingthe classical notion of tightness, we generalize this to arbitrary manifolds: We show that all tight simplicial 3-manifolds admit some perfect discrete Morse function. We also strengthen Chillingworth's theorem by proving that all convex simplicial 3-balls are non-evasive. In contrast, we show that many non-evasive 3-balls are not convex

5. A. Shabbir, **T. Zamfirescu**: *Gallai’s property for graphs in lattices on the torus and the Möbius strip*, Period. Math. Hungarica, online 09 noiembrie 2015; **ISI**, FI=0,479, SRI= 0,85.
We prove the existence of graphs with empty intersection of their longest paths or cycles as subgraphs of lattices on the torus and the Möbius strip.

Articles accepted in 2015

1. L. Yuan, C. T. Zamfirescu, T. I. **Zamfirescu**: Dissecting the square into five congruent parts, Discrete Math. 339 (2016), 288-298; **ISI**, FI=0,557, SRI= 0,765.
We give an affirmative answer to an old conjecture proposed by Ludwig Danzer: there is a unique dissection of the square into five congruent convex tiles.

2. **K. Adiprasito**, B. Benedetti: Subdivisions, shellability, and collapsibility of products, Combinatorica, to appear; **ISI**, FI=0,696, SRI= 2,16.

We prove that the second derived subdivision of any rectilinear triangulation of any convex polytope is shellable. Also, we prove that the first derived subdivision of every rectilinear triangulation of any convex 3-dimensional polytope is shellable. This complements Mary Ellen Rudin's classical example of a non-shellable rectilinear triangulation of the tetrahedron. Our main tool is a new relative notion of shellability that characterizes the behavior of shellable complexes under gluing. As a corollary, we obtain a new characterization of the PL property in terms of shellability: A triangulation of a sphere or of a ball is PL if and only if it becomes shellable after sufficiently many derived subdivisions. This improves on results by Whitehead, Zeeman and Glaser, and answers a question by Billera and Swartz. We also show that any contractible complex can be made collapsible by repeatedly taking products with an interval. This strengthens results by Dierker and Lickorish, and resolves a conjecture of Oliver. Finally, we give an example that this behavior extends to non-evasiveness, thereby answering a question of Welker.

3. **J. Rouyer**: *Steinhaus conditions for convex polyhedra,* to appear in K. Adiprasito et al. (eds.), Convexity and Discrete Geometry Including Graph Theory, Springer Proc. in Math. & Stat., (2016)

We show that any convex polyhedron has an open and dense set of points p admitting a unique antipode Fp, which in turn admits a unique antipode FFp, distinct from p.

4. N. Chevallier, A. Fruchard, **C. Vîlcu**: *Envelopes of α-sections*, to appear in K. Adiprasito et al. (eds.), Convexity and Discrete Geometry Including Graph Theory, Springer Proc. in Math. & Stat., (2016)

Let K be a planar convex body af area $|K|$, and take $0 < \alpha < 1$. An α-section of K is a line cutting K into two parts, one of which has area $\alpha|K|$. This article presents a systematic study of the envelope of α-sectionsand its dependence on α.

Talks at international conferences and workshops

1. T. Zamfirescu, Discs and other miscreants held in cages, 5th International Conference on Combinatorics, Graph Theory, and Applications, Elgersburg, Germany, March 2015

2. C. Vîlcu, *On the envelope of α-sections of a convex body*, 5th Workshop for Young Researchers in Mathematics, Constanța, May 2015.

3. C. Vîlcu, *Baire categories for Alexandrov surfaces*, The Eighth Congress of Romanian Mathematicians, Iași, June 2015.

Project Director,
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