FMI, CS, Master I
Winter Term 2014/2015
Techniques of Combinatorial Optimization
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Seminar 1

(S1.1) A subset $P \subseteq \mathbb{R}^n$ is a polyhedron if and only if $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ for some matrix $A \in \mathbb{R}^{m \times n}$ and some vector $b \in \mathbb{R}^m$.

(S1.2) Prove that

(i) Affine sets are polyhedra.

(ii) Singletons are polyhedra of dimension 0.

(iii) Lines are polyhedra of dimension 1.

(iv) The unit cube $C_3 = \{x \in \mathbb{R}^n \mid 0 \leq x_i \leq 1 \text{ for all } i = 1, 2, 3\}$ in $\mathbb{R}^3$ is a full-dimensional polyhedron.

(S1.3) [Farkas lemma - variant] The system $Ax = b$ has a solution $x \geq \mathbf{0}$ if and only if $y^T b \geq 0$ for each $y \in \mathbb{R}^m$ with $y^T A \geq \mathbf{0}^T$.

(S1.4) Let (P) and (D) be the primal and dual LPs.

(i) If both (P) and (D) are feasible, then they are bounded.

(ii) If either (P) or (D) is unfeasible, then the other is either unfeasible or unbounded.

(iii) If either (P) or (D) is unbounded, then the other is unfeasible.

(iv) If either (P) or (D) is bounded, then the other is bounded too.