

Seminar 4

(S4.1) Let G be the complete graph K_3 on three vertices. Prove that its incidence matrix is not totally unimodular.

(S4.2) Let $A \in \mathbb{R}^{m \times n}$ be a TU matrix, let $b \in \mathbb{Z}^m$ and let $c \in \mathbb{Z}^n$. Then both optima in the LP duality equation

$$\max\{c^T x \mid x \geq \mathbf{0}, Ax \leq b\} = \min\{y^T b \mid y \geq \mathbf{0}, y^T A \geq c^T\}.$$

have integer optimal solutions (if the optima are finite).

(S4.3) Let $G = (V, E)$ be a bipartite graph and $w : E \rightarrow \mathbb{N}$ be a weight function. The maximum weight of a matching in G is equal to the minimum value of $\sum_{v \in V} y_v$, where y ranges over all functions $y : V \rightarrow \mathbb{N}$ such that $y_u + y_v \geq w(e)$ for each edge $e = uv$ of G .

(S4.4) Let A be the incidence matrix of a cycle of length 5. Prove that A is a square matrix and compute its determinant.

(S4.5) Give examples of nonempty polyhedra $P \subseteq \mathbb{R}^n$ such that

(i) $P \cap \mathbb{Z}^n = \emptyset$, hence $P_I = \emptyset$.

(ii) $P \cap \mathbb{Z}^n \neq \emptyset$ and for every $c \in \mathbb{R}^n \setminus \{0\}$, $\max\{c^T x \mid x \in P \cap \mathbb{Z}^n\} < \max\{c^T x \mid x \in P\}$.