

Seminar 6

In the sequel, $D = (V, A)$ is a digraph, $s, t \in V$.

(S6.1) Assume that $f_1, \dots, f_n : A \rightarrow \mathbb{R}$ are mappings satisfying the flow conservation law at $v \in V$. Then any linear combination of f_1, \dots, f_n satisfies the flow conservation law at v .

(S6.2) Prove Proposition 3.4.2.

(S6.3) For any s - t path P in D , prove that χ^P satisfies the flow conservation law at every $v \neq s, t$ and that $\text{value}(\chi^P) = 1$.

(S6.4) Let $N = (D, s, t)$ be a unit capacity network, $k \geq 1$ and P_1, \dots, P_k be k arc-disjoint s - t paths in D . Then for all $k \geq 1$,

$$f := \chi^{P_1} + \dots + \chi^{P_k}$$

is an s - t $\{0, 1\}$ -flow f with $\text{value}(f) = k$.

(S6.5) Let us recall that a subset $B \subseteq A$ is said to be an s - t *disconnecting arc set* if B intersects each s - t path. Prove that each s - t cut is an s - t disconnecting arc set.