FMI, CS, Master I
Winter Term 2014/2015
Techniques of Combinatorial Optimization
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Seminar 7

(S7.1) Give an example where feasible circulations do not exist.

(S7.2) Prove Menger's theorem (directed vertex-disjoint version): If $D = (V, A)$ is a digraph and $S, T \subseteq V$, then the maximum number of vertex-disjoint S-T-paths is equal to the minimum size of an S-T disconnecting vertex set.
(This is Theorem 4.1.10 in the lecture notes).

(S7.3) Let $D = (V, A)$ and f', f be feasible circulations in D. Define $g : A \cup A^{-1} \to \mathbb{R}$ as follows: for all $a \in A$,

$$g(a) = \max\{0, f'(a) - f(a)\}, \quad g(a^{-1}) = \max\{0, f(a) - f'(a)\}.$$

Prove that

(i) g is a circulation in $\overline{D}$;

(ii) $\mathrm{cost}(g) = \mathrm{cost}(f') - \mathrm{cost}(f)$;

(iii) $g(e) = 0$ for all $e \notin A(D_f)$.

(This is Lemma 3.6.3 in the lecture notes).