SNSB **HW #4**
Topics in Galois and Cogalois Theory **Spring 2011**
 March 17, 2011

1. Prove that $\mu_n(F(X_1,\ldots,X_m)) = \mu_n(F)$ for any field F and any m, $n \in \mathbb{N}^*$.

2. Let p be any positive prime number, and let n be any positive integer. Prove that the extension $\mathbb{F}_p(X_1,\ldots,X_n)/\mathbb{F}_p(X_1^p,\ldots,X_n^p)$ is an extension of degree p^n, with Cogalois group isomorphic to a countably infinite direct sum of copies of the cyclic group $\mathbb{Z}_p$.

3. Show that $\mathrm{Cog}(\mathbb{F}_4/\mathbb{F}_2) \cong \mathbb{F}_4^*$.

4. Prove that for any extension $\mathbb{F}_{q^n}/\mathbb{F}_q$ of finite fields, one has

$$\mathrm{Cog}(\mathbb{F}_{q^n}/\mathbb{F}_q) \cong \mathbb{Z}_m,$$

where $m = 1 + q + \ldots + q^{n-1}$.

5. Let $E = \mathbb{F}_2(X)$ and $F = \mathbb{F}_2(X^2)$. Show that the extension E/F is simultaneously $F^*\langle X \rangle$-Kneser and $F^*\langle X+1 \rangle$-Kneser, but the groups $F^*\langle X \rangle$ and $F^*\langle X+1 \rangle$ are distinct.

6. Prove that any quadratic extension E/F with $\mathrm{Char}(F) \neq 2$ is Kneser. Is the result true in case $\mathrm{Char}(F) = 2$?

7. Prove that the extension $\mathbb{F}_4/\mathbb{F}_2$ is not Kneser.

8. Investigate whether or not the quotient extension of a Kneser extension is also Kneser.

9. Is the compositum of two Kneser (resp. radical) extensions again a Kneser (resp. radical) extension?

10. Give an example of a radical extension which is not a Kneser extension.